Summer 7-31-2013

# Reliability Analysis of Shallow Foundations Bearing Capacity on Sand

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RELIABILITY ANALYSIS OF SHALLOW FOUNDATIONS BEARING CAPACITY ON SAND

BY

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A THESIS

PRESENTED TO THE FACULTY OF

THE GRADUATE COLLEGE AT THE UNIVERSITY OF NEBRASKA

IN PARTIAL FULFILLMEN OF REQUIREMENTS

FOR THE DEGREE OF MASTER OF SCIENCE

MAJOR: CIVIL ENGINEERING

UNDER THE SUPERVISION OF PROFESSOR MARIA SZERSZEN

LINCOLN, NEBRASKA

JULY, 2013

RELIABILITY ANALYSIS OF SHALLOW FOUNDATIONS

BEARING CAPACITY ON SAND

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University of Nebraska, 2013

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Bearing capacity of shallow foundation is one of the most challenging problems for engineers. The difficulty comes from multiple sources of variability and uncertainty. There is an uncertainty in live load. Soil properties including: unit weight, cohesion, and angle of friction represent sources of variability in the determining bearing capacity. The current theories used in practice only estimate bearing capacity and does not give an exact value for it because of these sources of variability. Currently, there are Terzaghi, Meyerhof, Vesic, and Hansen theories for dealing with this problem. Based on previous research Terzaghi theory was found to be the most close estimation tool to the real value of bearing capacity.

The aim of this paper is to calculate the reliability index of Terzaghi's theory and to propose a resistance factor that corresponds a reliability index of 4. The reliability analysis was done for circular and square footing. Loads, soil properties, width, and depth

of the foundation were considered random variables to get a complete picture of the bearing capacity problem. The reliability analysis was done using Monte Carlo simulation and the First order Second Moment method to calculate the reliability index.

# DEDICATION

TO THE SAVIOR OF THE WORLD, IMAM MAHDI (A.S.S)

# ACKNOWLEDGMENTS

My most sincere appreciation goes to my professor Dr. Maria Szerszen for her valuable help, guidance, and encouragement through my work on this topic.

I would like to offer my deepest gratitude and appreciation for my parent for their unconditional love and care. I owe this achievement to my dad as he always been the light in darkness, the reason to push forward, and the guide for my journey in life. Without my parents blessing and prayers I would have been lost trying to find a safe shore to land.

Special thanks for my younger brothers Ahmad and Mustafa for their support and help in time of need as they served as my right and left hand.

Thanks for my wife for her patience, love, and for being my partner in my journey. Thanks for my son for being understanding and quite while writing this paper.

Also, I would to offer my thanks for my friends: Mahdi Alhajji, Mohammed Haft Javahria, and Hooman Ghasmi for their friendship and respect.

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Based on ASD method, which is currently adapted for geotechnical design, Terzaghi equation produce a probability of failure in the range of (0.0046-0.00248) for square

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# CHAPTER 1. INTRODUCTION

## 1.1 BEARNING CAPACITY CALCULATION METHODS

Solution for mechanics problems must satisfy three conditions of equilibrium, compatibility, and material property. These conditions are sufficient to determine the distribution of stress and displacement up to the moment of a collapse. However, a complete solution is very difficult because it requires the knowledge of soil behavior under the past and future loads. Therefore in solving geotechnical problems scientists try to answer two questions. The first question is what are the structural displacements under the present working loads? The second question is the working load less than the collapse load? (Cernica, 1995). In order to simplify the process some of the equilibrium and compatibility conditions can be ignored. When ignoring the equilibrium conditions the upper bound to the ultimate load can be calculated and by ignoring the compatibility conditions a lower bound to the ultimate load is calculated as well. The main feature of the upper and lower bound is they will bracket the real ultimate load.

The lower and upper bound theorem are fundamental principles of plasticity that would provide a method to calculate the ultimate load for materials that have perfectly plastic behavior. A material with perfectly plastic behavior means that it will strain at a constant rate at failure with an associated flow rule. Figure (1) shows that behavior of a perfectly plastic soil behavior.

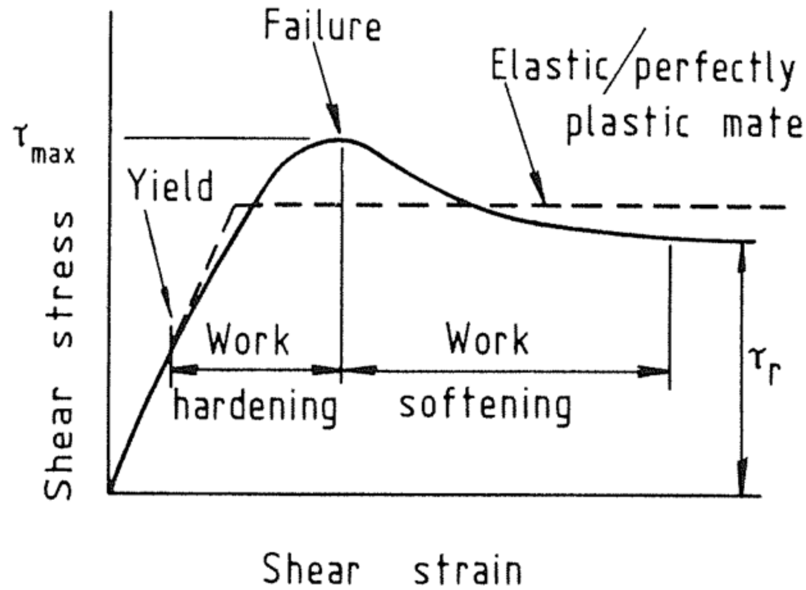



Figure 1.1: Schematic of perfectly plastic soil behavior ( Rao, 2011).

An associated flow rule means that the plastic potential envelope that is the same as the failure envelope as shown in figure 2.

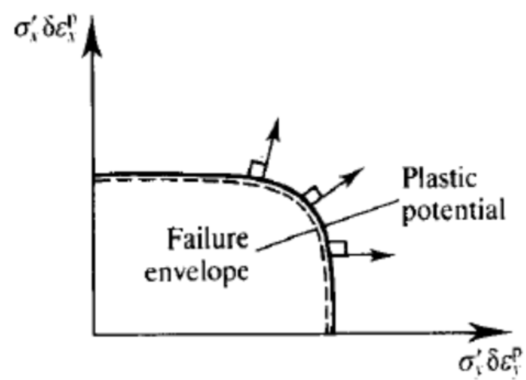



Figure 1.2: Schematic of the associate flow rule ( Rao, 2011).

These two principles are advantageous for geotechnical engineering because they allow to solve complex soil problems with relative ease. The complexity is related to solving nonlinear stress strain relationship using constitutive models where some other engineering problems can be simplified to one dimension. However, the limit theorem makes the solution for these challenging geotechnical problems possible by considering only the shear strength as compare to a complete stress strain behavior. Moreover, this theorem presents a way to check the accuracy of the ultimate load because it provides engineers with a lower value and an upper value for the collapse load.

Another method that is used to estimate bearing capacity problem is Slip Line method. This method describes a plastic equilibrium stress field beneath the foundation that is not necessarily extended to a satisfactory distance (Cernica, 1995). Also, it combines Coulomb criterion with equilibrium equations to obtain a set of differential equations. These differential equations describe the plastic region beneath the foundation that is not extended long enough. Therefore, the solution obtained from this method is not always the true solution (Chen, 2007). In order to obtain a true solution an associated flow rule along with an extension of the stress is required to obtain this solution.

The third method is the Limit Equilibrium Method. This method can be described as an approximation for the bearing capacity problem. It is based on stress distribution assumption that would simplify the problem which makes it possible to obtain an approximate solution.

There is no method of calculating exactly the ultimate bearing capacity of soil. All bearing capacity theories are just an estimation tool (Bowel, 1996). Currently, there are four bearing capacity theories. Terzaghi's theory is the first one. Meyerhof's theory is the second one. There is Hansen and Vesic theories as well. Research has shown that Terzaghi's theory produce the closest value to the actual bearing capacity. Due to the uncertainty in soil properties, it is very important to study the reliability of the current design practice to ensure the safety of structures. Therefore, the aim of the work is to calculate the reliability of shallow foundations bearing according to Terzaghi's theory. The second goal is to propose a resistance factor value that corresponds to a reliability index of 4 for LRFD design.

Different modes of bearing capacity factors will be considered as well to provide a clear view of the bearing capacity of soils. In the last section of the paper a brief literature review will be provided to show the latest information obtained regarding the bearing capacity of different soils. Lastly, an example of calculating the bearing capacity will be presented using different models to show the model that would produce the most reliable results.

## 1.2 OVERVIEW OF THE BEARING CAPACITY THEORIES.

The first model is the Prandtl (1920) model. Prandtl studied the problem of bearing capacity using plastic equilibrium method with Mohr Coloumb failure criterion. His theory reflects on the penetration process of a hard object into soft, homogenous, and isotropic material. In his study, he formulated a two dimensional infinitely long punch onto a horizontal surface. The punch in Prandtl's theory can be modeled as a uniformly stressed strip foundation of width B. The soil beneath this strip foundation is considered to be the softer material. Figure 3 shows a schematic of Prandtl's theory.

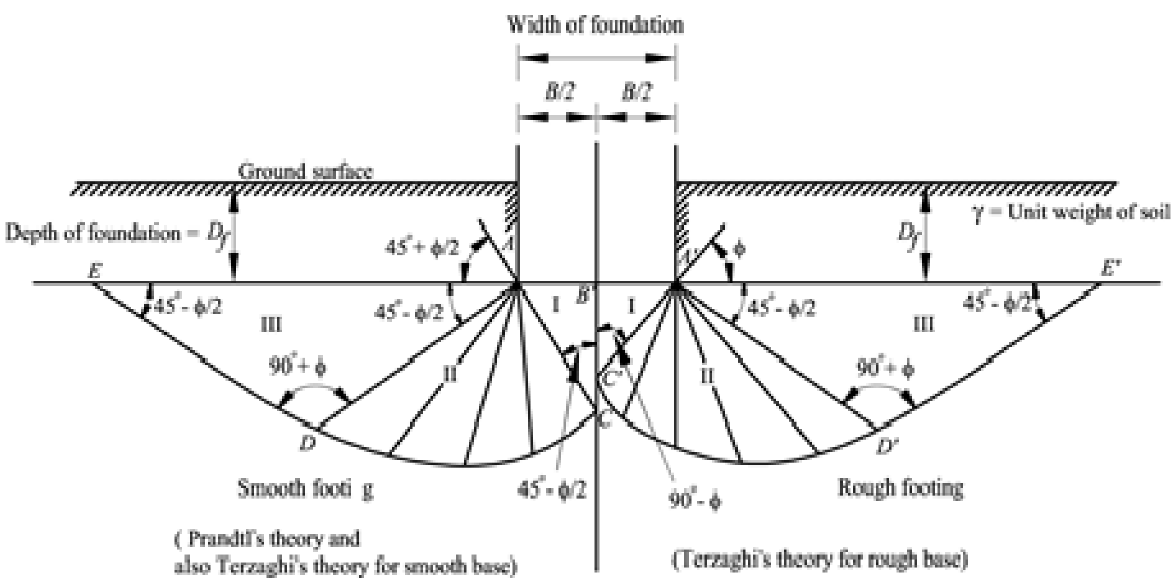



Figure1.3: Prandtl's theory of plastic equilibrium (Cernica, 1995)

This figure shows three zones developed in the soil:

1. Zone I : the soil wedge ABC is assumed to be weightless and in an active Rankine state and it will move downward as a unit.
2. Zone II : the soil wedge ACD is the radial shear zone. It is assumed to be in state of radial plastic flow and the boundary as a logarithmic spiral with the center being at A.
3. Zone III : the soil wedge ADE is the Rankine passive zone. It is assumed to be forced by a passive pressure upward and outward as a unit.

Moreover, Prandtl assumed the angle between the punch and the soil wedge under the footing that is the angle BAC, to be 45+ θ/2. He obtained a second order differential equation for which the solution is the analytical expression for the ultimate bearing capacity:

$$q_u = \left(\frac{c}{\tan \phi} + \frac{1}{2}\gamma B\sqrt{K_p}\right)(K_p e^{\pi \tan \phi} - 1)$$

(Rao, 2011)

$\gamma =$ unit weight of the soil.

where

c, $\phi =$ shear strength parameters of soil that is, cohesion and angle of internal friction

$$K_p = \text{Rankine's passive earth pressure coefficient} = \frac{1+\sin\phi}{1-\sin\phi}$$

Prandtl theory was originally derived for a weightless soil and a smooth foundation base, and the term $\frac{1}{2}\gamma B\sqrt{K_p}$ was added later by Taylor to account for the shear strength caused by the overburden pressure of the soil. Prnadlt's theory was the most accurate way of calculated bearing capacity but it was a start for this complicated problems. Some of the assumption of Prandtl's theory include the assumption of isotropic and homogenous soil, the infinitely long footing, and the smooth interface between the footing and the soil. These assumptions don't compley with practical design applications, which point out an important deficincies in Prandlt theory of bearing capacity. These deficincies have let other researchers to make some modification to Prandtl's theory. These researchers include Terzaghi, Meyerhof, and Hansen.

The second model is Terzaghi's bearing capacity equation.Terzaghi's equation is based on pervious work of Prandtl with some modifications. Terzaghi defined a foundation to be shallow when the depth of the foundation is less than or equal to the width of the foundation Df/B $\leq$ 1 (Cerato, 2005). In his equation, Terzaghi made some assumptions regarding the footing soil system. These assumptions include:

1. The footing base is rough to account for the friction between the base and soil

2. The weight of the soil above the base of the footing is considered to be uniformly applied as a surcharge and has no shear strength.
3. Soil cohesion is considered in cohesive soil.
4. The shear resistance above the base of the footing is not considered.
5. The general shape of the wedges in Prandtl's theory is not changed.
6. The general share mode of failure governs.
7. The applied load is considered to be vertical to the centroid of the foundation.
8. The foundation is considered to be rigid in comparison to the soil undernathe it.
9. The angle between the triangular wedge and the horizantal is θ instead of θ/2+45 as assumed by Prandtl.

Based on these assumptions, Terzaghi presented his equation for ultimate bearing capacity of strip shallow foundation.

$$q_{ult} = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma$$

Where

q= vertical effective stress = $\gamma D_f$

c = choesion of the soil

$\gamma = unit\ weight\ of\ the\ soil$

$D_f\ = depth\ of\ the\ footing$

B = width of the footing

$N_y = bearing\ capacity\ factor\ due\ to\ the\ weight\ of\ the\ soil$

$N_q = bearing\ capacity\ factor\ for\ the\ surcharge\ load$

$N_c = bearing\ capacity\ factor\ for\ the\ cohesion\ of\ the\ soil$

Where

$$\boxed{N_q = \frac{a_\theta^2}{2\cos^2(45 + \phi'/2)}}$$

$$a_\theta = e^{\pi(0.75 - \phi'/360)\tan\phi'}$$

$$\boxed{N_c = \frac{N_q - 1}{\tan\phi'}}$$

$$\boxed{N_\gamma \approx \frac{2\,(N_q + 1)\tan\phi'}{1 + 0.4\sin(4\,\phi')}}$$

(after Coduto, 2001)

Where

$K_{p\gamma} = passive\ earth\ pressure\ coefficient$

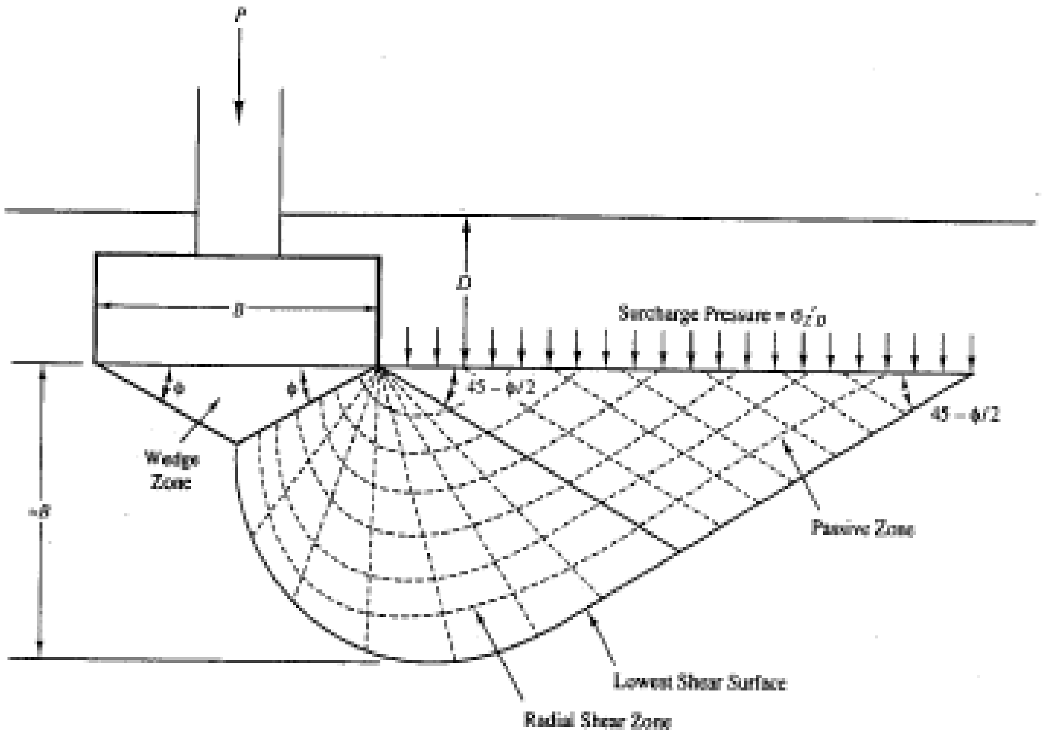



Figure 1.4: Geometry of Terzaghi's failure surface (Coduto, 2001)

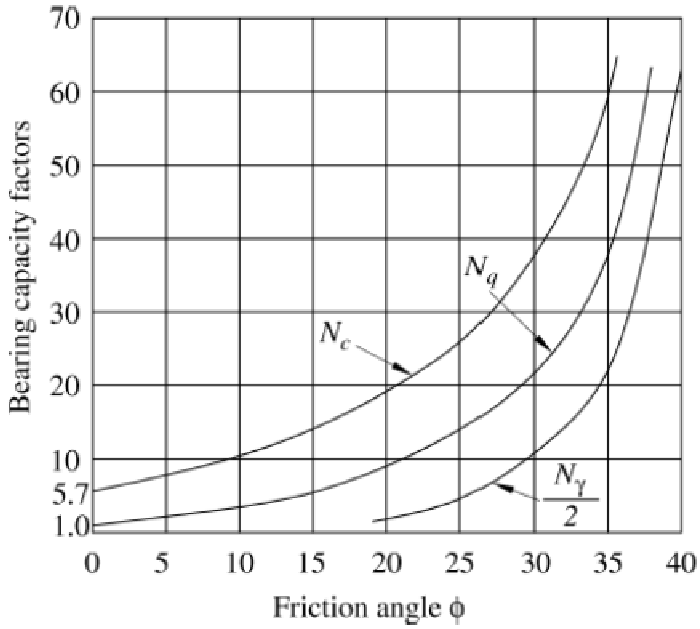
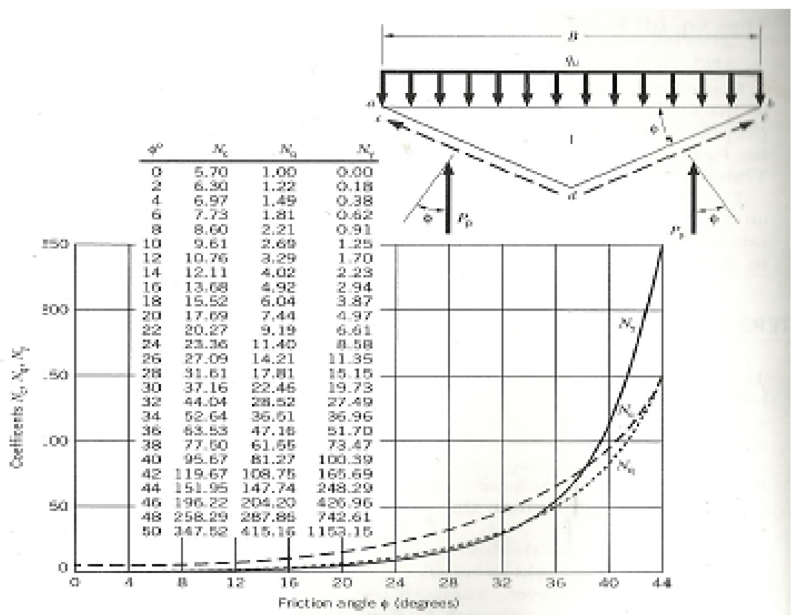



Figure 1.5: Terzaghi's bearing capacity factors for general shear failure (Rao, 2011)



| $\phi^\circ$ | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|
| 0 | 5.70 | 1.00 | 0.00 |
| 2 | 6.30 | 1.22 | 0.18 |
| 4 | 6.97 | 1.49 | 0.38 |
| 6 | 7.73 | 1.81 | 0.62 |
| 8 | 8.60 | 2.21 | 0.91 |
| 10 | 9.61 | 2.69 | 1.25 |
| 12 | 10.76 | 3.29 | 1.70 |
| 14 | 12.11 | 4.02 | 2.23 |
| 16 | 13.68 | 4.92 | 2.94 |
| 18 | 15.52 | 6.04 | 3.87 |
| 20 | 17.69 | 7.44 | 4.97 |
| 22 | 20.27 | 9.19 | 6.61 |
| 24 | 23.36 | 11.40 | 8.58 |
| 26 | 27.09 | 14.21 | 11.35 |
| 28 | 31.61 | 17.81 | 15.15 |
| 30 | 37.16 | 22.46 | 19.73 |
| 32 | 44.04 | 28.52 | 27.49 |
| 34 | 52.64 | 36.51 | 36.96 |
| 36 | 63.53 | 47.16 | 51.70 |
| 38 | 77.50 | 61.55 | 73.47 |
| 40 | 95.67 | 81.27 | 100.39 |
| 42 | 119.67 | 108.75 | 165.69 |
| 44 | 151.95 | 147.74 | 248.29 |
| 46 | 196.22 | 204.20 | 426.96 |
| 48 | 258.29 | 287.86 | 742.61 |
| 50 | 347.52 | 415.16 | 1153.15 |



Figure 1.6: Terzaghi's bearing capacity factors along with the penetrating wedge

(Cernica, 1995)

The third model is based on Meyerhof's theory of bearing capacity. Meyerhof proposed a bearing capacity equation similar to Terzaghi's equation but he added shape factor s, depth factor d, and inclination factor i. Meyerhof included these factors to account for:

1. Rectangular footing
2. Load inclination
3. Shear resistance in the failure surface in the soil above the base of the footing.

His general equation is :

$$q_u = cN_c s_c d_c i_c + q'_o N_q s_q d_q i_q + \frac{1}{2}\gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$ (Murthy,2011)

Where

c = unit cohesion

$q_0^` = effective\ overburden\ pressure = \gamma D_f$

$\gamma^` = effective\ unit\ weight\ of\ the\ soil\ below\ the\ foundation\ base$

$D_f = depth\ of\ the\ foundation$

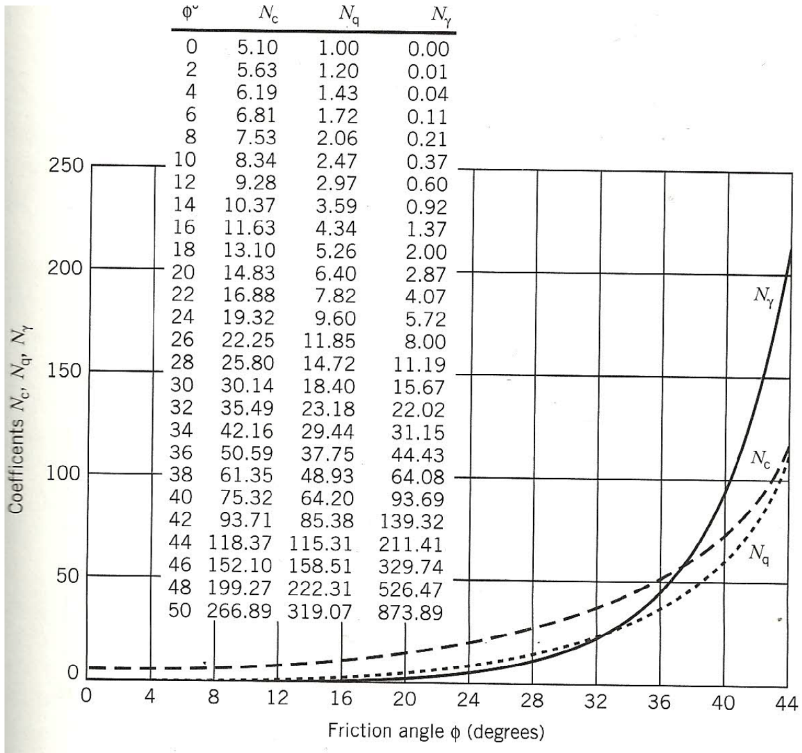
$S_{c\,,}\, S_{q\,,}\, S_\gamma = shape\ factors$

$d_{c\,,} d_q, d_\gamma = depth\ factors$

$i_{c\,,} i_q\,, i_\gamma = load\ inclination\ factors$

B =width of the foundation

$N_c, N_q, N_\gamma = bearing\ capacity\ factors$




| φ° | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|
| 0 | 5.10 | 1.00 | 0.00 |
| 2 | 5.63 | 1.20 | 0.01 |
| 4 | 6.19 | 1.43 | 0.04 |
| 6 | 6.81 | 1.72 | 0.11 |
| 8 | 7.53 | 2.06 | 0.21 |
| 10 | 8.34 | 2.47 | 0.37 |
| 12 | 9.28 | 2.97 | 0.60 |
| 14 | 10.37 | 3.59 | 0.92 |
| 16 | 11.63 | 4.34 | 1.37 |
| 18 | 13.10 | 5.26 | 2.00 |
| 20 | 14.83 | 6.40 | 2.87 |
| 22 | 16.88 | 7.82 | 4.07 |
| 24 | 19.32 | 9.60 | 5.72 |
| 26 | 22.25 | 11.85 | 8.00 |
| 28 | 25.80 | 14.72 | 11.19 |
| 30 | 30.14 | 18.40 | 15.67 |
| 32 | 35.49 | 23.18 | 22.02 |
| 34 | 42.16 | 29.44 | 31.15 |
| 36 | 50.59 | 37.75 | 44.43 |
| 38 | 61.35 | 48.93 | 64.08 |
| 40 | 75.32 | 64.20 | 93.69 |
| 42 | 93.71 | 85.38 | 139.32 |
| 44 | 118.37 | 115.31 | 211.41 |
| 46 | 152.10 | 158.51 | 329.74 |
| 48 | 199.27 | 222.31 | 526.47 |
| 50 | 266.89 | 319.07 | 873.89 |

Figure 1.7: Meyerhof's bearing capacity coefficients (Cernica, 1995)

The fourth model is based on Hansen & Vesic equation of bearing capacity. Hansen equation is an extension of Meyerhof's equation. The $N_c, N_q$, are the same as Meyerhof's. The $N_\gamma$ is the same as Meyerhof's up to angle of friction value of 35 degrees ( Cernica,1995). There are some differences for the higher value of angel of friction. However, Hansen's values are more conservative that Meyerhof's (Cernica, 1995). Hansen included factors of shape, depth, and load inclination as well as ground factors and base factor for footing on a slope. His equation is:

$$q_{ult} = C^{`}\ N_c\ s_c\ d_c\ i_c\ b_c\ g_c + q_0^{`}\ N_q\ s_q\ d_q\ i_q\ b_q\ g_q + 0.5\ B\ \gamma^{`}\ N_\gamma\ s_\gamma\ d_\gamma\ i_\gamma\ b_\gamma\ g_\gamma$$

Where

c = unit cohesion

$q_0^{`} = effective\ overburden\ pressure$ = γD

$\gamma^{`} = effective\ unit\ weight\ of\ the\ soil\ below\ the\ foundation\ base$

$D_f = depth\ of\ the\ foundation$

$S_c, S_q, S_\gamma = shape\ factors$

$d_c, d_q, d_\gamma = depth\ factors$

$i_c, i_q, i_\gamma = load\ inclination\ factors$

$b_c\ b_q\ b_\gamma =$ base inclination factors

$g_c\ g_q\ g_\gamma =$ ground inclination factors

| φ° | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|
| 0 | 5.10 | 1.00 | 0.00 |
| 2 | 5.63 | 1.20 | 0.01 |
| 4 | 6.19 | 1.43 | 0.05 |
| 6 | 6.81 | 1.72 | 0.11 |
| 8 | 7.53 | 2.06 | 0.22 |
| 10 | 8.34 | 2.47 | 0.39 |
| 12 | 9.28 | 2.97 | 0.63 |
| 14 | 10.37 | 3.59 | 0.97 |
| 16 | 11.63 | 4.34 | 1.43 |
| 18 | 13.10 | 5.26 | 2.08 |
| 20 | 14.83 | 6.40 | 2.95 |
| 22 | 16.88 | 7.82 | 4.13 |
| 24 | 19.32 | 9.60 | 5.75 |
| 26 | 22.25 | 11.85 | 7.94 |
| 28 | 25.80 | 14.72 | 10.94 |
| 30 | 30.14 | 18.40 | 15.07 |
| 32 | 35.49 | 23.18 | 20.79 |
| 34 | 42.16 | 29.44 | 28.77 |
| 36 | 50.59 | 37.75 | 40.05 |
| 38 | 61.35 | 48.93 | 56.18 |
| 40 | 75.32 | 64.20 | 79.54 |
| 42 | 93.71 | 85.38 | 113.96 |
| 44 | 118.37 | 115.31 | 165.58 |
| 46 | 152.10 | 158.51 | 244.65 |
| 48 | 199.27 | 222.31 | 368.68 |
| 50 | 266.89 | 319.07 | 568.59 |

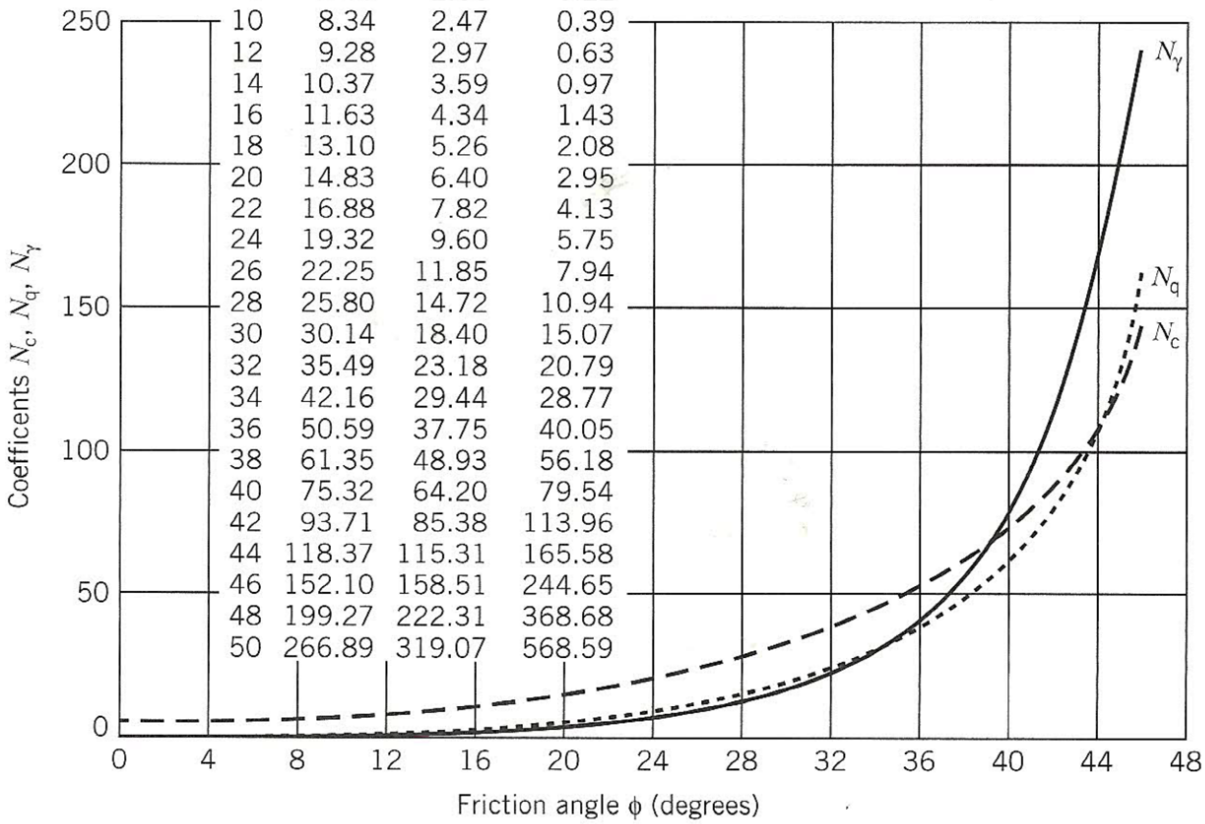



Figure1.8: Hansen's bearing capacity coefficients.

Vesic equation is the same as Hansen's. The only difference is in the values of $N_\gamma$ which are higher than Hansen's for angle of frction value of less than 40 degrees and lower for values higher than 45 degrees (Cernica, 1995). Table 1 shows the difference in Meyerhof's, Hansen's, and Vesic's factors as presented by Murth, 2011.

| Factors | Meyerhof | Hansen | Vesic |
|---|---|---|---|
| $s_c$ | $1+0.2N_\phi \frac{B}{L}$ | $1+\frac{N_q}{N_c}\frac{B}{L}$ | |
| $s_q$ | $1+0.1N_\phi \frac{B}{L}$ for $\phi > 10^\circ$ | $1+\frac{B}{L}\tan\phi$ | |
| $s_\gamma$ | $s_\gamma = s_q$ for $\phi > 10^\circ$<br>$s_\gamma = s_q = 1$ for $\phi = 0$ | $1-0.4\frac{B}{L}$ | The shape and depth factors of Vesic are the same as those of Hansen. |
| $d_c$ | $1+0.2\sqrt{N_\phi}\frac{D_f}{B}$ | $1+0.4\frac{D_f}{B}$ | |
| $d_q$ | $1+0.1\sqrt{N_\phi}\frac{D_f}{B}$ for $\phi > 10^\circ$ | $1+2\tan\phi(1-\sin\phi)^2\frac{D_f}{B}$ | |
| $d_\gamma$ | $d_\gamma = d_q$ for $\phi > 10^\circ$<br>$d_\gamma = d_q = 1$ for $\phi = 0$ | 1 for all $\phi$<br>Note; Vesic's $s$ and $d$ factors = Hansen's $s$ and $d$ factors | |
| $i_c$ | ${1-\frac{\alpha^\circ}{90}}^{2}$ for any $\phi$ | $i_q-\frac{1-i_q}{N_q-1}$ for $\phi > 0$ | Same as Hansen for $\phi > 0$ |
| | | $0.5\ {1-\frac{Q_h}{A_f c_a}}^{\frac{1}{2}}$ for $\phi = 0$ | $1-\frac{mQ_h}{A_f c_a N_c}$ |
| $i_q$ | $i_q = i_c$ for any $\phi$ | ${1-\frac{0.5Q_h}{Q_u + A_f c_a \cot\phi}}^{5}$ | ${1-\frac{Q_h}{Q_u + A_f c_a \cot\phi}}^{m}$ |
| $i_\gamma$ | ${1-\frac{\alpha^\circ}{\phi^\circ}}^{2}$ for $\phi > 0$<br>$i_\gamma = 0$ for $\phi = 0$ | ${1-\frac{0.7Q_h}{Q_u + A_f c_a \cot\phi}}^{5}$ | ${1-\frac{Q_h}{Q_u + A_f c_a \cot\phi}}^{m+1}$ |

$$N_\gamma = (N_q - 1)\tan(1.4\phi) \qquad \text{(Meyerhof)}$$

$$N_\gamma = 1.5(N_q - 1)\tan\phi \qquad \text{(Hansen)}$$

$$N_\gamma = 2(N_q + 1)\tan\phi \qquad \text{(Vesic)}$$

Table 1: Meyerhof, Hansen, and Vesic bearing capacity factors (Cernica, 1995).

## 1.3 BEARING CAPACITY FAILURE MODELS

Researchers have shown that bearing capacity failure happens due to shear failure of the soil beneath the footing. They have observed three predominant failure types. The first one is the general shear failure (a). The second one the local shear failure (b) and the third one is the punching shear failure.

The general shear failure happens in dense sand of $D_r > 70\%$ and in saturated normally consolidated clays ( Coduto, 2001). This type of failure is sudden and happens when the settlement reaches 7 % of the foundation width (Coduto, 2001). When this type of failure happens a clear bulge appears on the ground surface near the foundation. This is the most common type of failure.

The second type is the local shear failure which happens in medium dense sand that has a relative density between 70% and 35% (Coduto, 2001). This type of failure is not sudden and happens when the settlement exceeds 8% of the foundation width. The failure surface will gradually extend outward from the foundation but a sudden failure may not ever happen and the foundation will continue to sink into the soil (Coduto, 2001).

The third kind of failure is the punching shear failure. This type of failure happens in loose sands of relative density of less than 35%. In this type of failure the settlement will be between 15% to 20% of the foundation width. Bulging may never happen and the failure surface which is a vertical and follows the perimeter of the foundation and it will never reach the ground surface. The figure below shows the three types of failure.

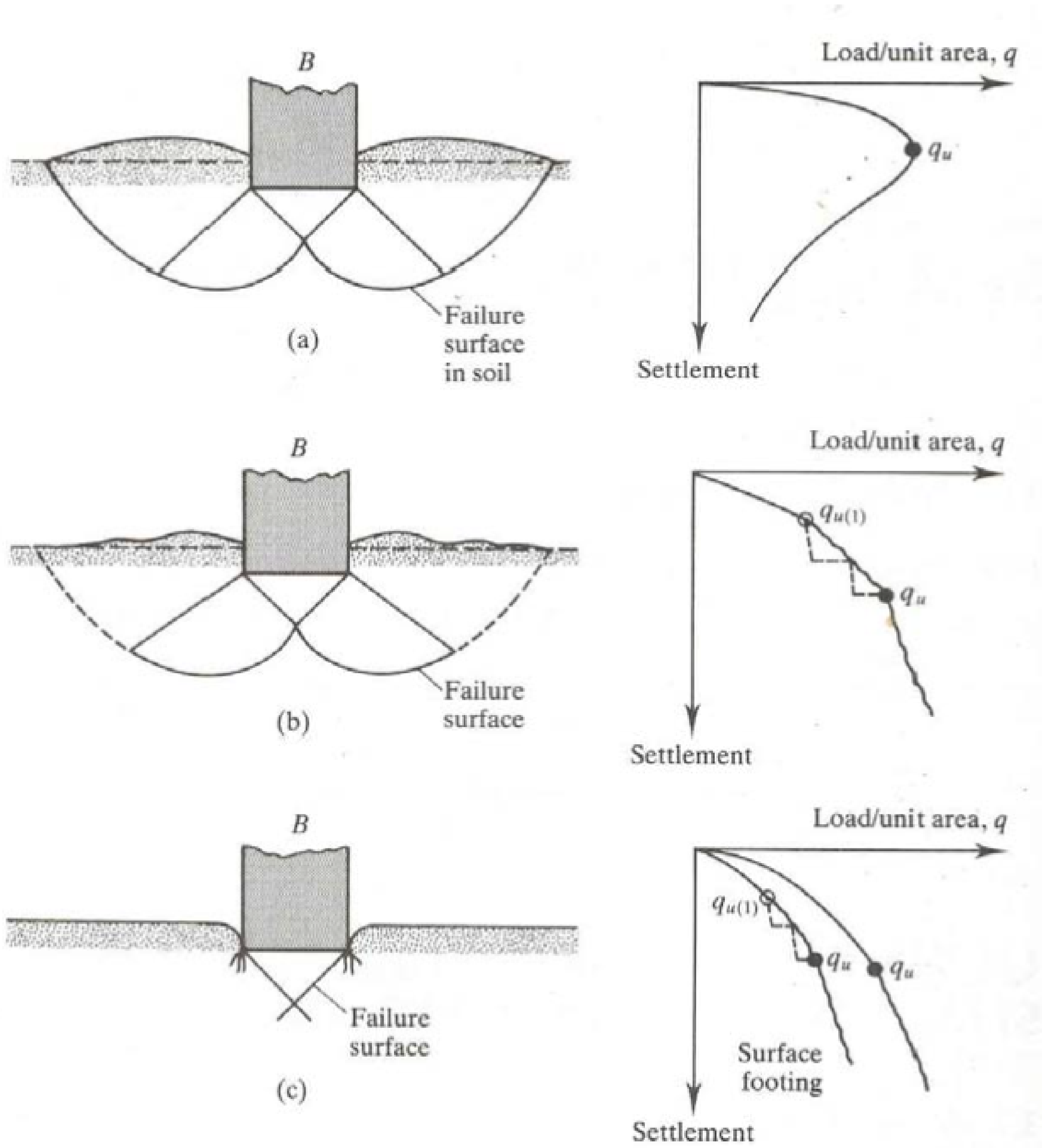



Figure 1.9: Bearing capacity failure modes (Das, 2007)

These types of failure were observed by Vesic (1963) during tests on model footings. It should be noted here that these modes are centrically loaded footings. Any eccentricity in the loads will change the failure mode and the foundation will tilt in the direction of eccentricity. The reason for tilting is due to the variation of shear strength and compressibility of the soil from one point to another and this would cause a larger yielding on one side of the foundation. This would throw the loads center of gravity off center toward the tilted side and would cause even a greater yielding (Murthy, 2011). The figure below shows the failure mode as the relative density of sand changes along with the relative depth of foundation as it was observed by Vesic (1963).

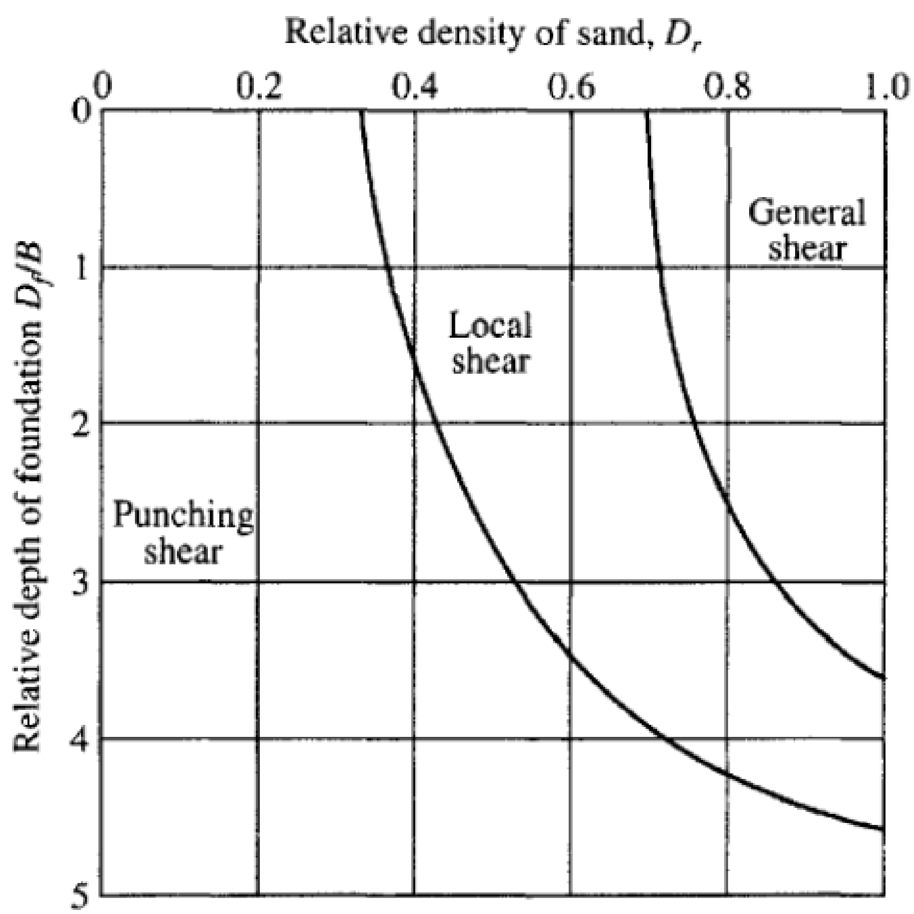



Figure 1.10: Bearing capacity failure modes based on model footing tests of Vesic (1963)

## 1.4 GROUND WATER EFFECT ON BEARING CAPACITY

The equations that have been developed to estimate the bearing capacity of soils are based on the assumption that the ground water table is located well below the foundation. When exploring the subsurface condition, the ground water table level must be determined because it will have a great effect on the bearing capacity of the soil. The water table affect the shear strength of the soil in two ways. The first way is the reduction of the apparent cohesion and the second way is the increase in the pore water pressure. There are three cases that must be addressed when determining the bearing capacity in the presence of ground water table.

$q_{ult} = C^{`} N_c + \sigma^{`}_{zD} N_q + 0.5\, \gamma^{`} B N_\gamma$ (Terzaghi's bearing capacity equation (Coduto, 2001))

**Case I**: $D_w \leq$ D

$$\gamma^{`} = \gamma_b = \gamma - \gamma_w$$

**Case II**: $D < D_w < D + B$

$$\gamma^{`} = \gamma - \gamma_w \left(1 - \left(\frac{D_w - D}{B}\right)\right)$$

**Case III** : D + B ≤ $D_w$

$$\gamma^{`} = \gamma$$

Where

D = depth of embedment

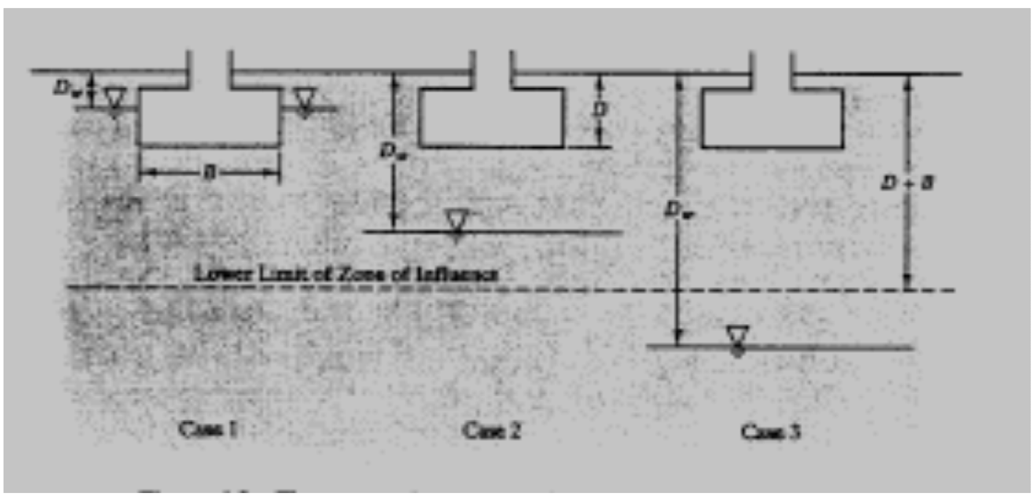



Figure 1.11: Three groundwater cases for bearing capacity analysis (Coduto, 2001)

# CHAPTER 2. REVIEW OF TECHNICAL LITERATURE

Most of the research done on the subject of bearing capacity has used Terzaghi's equation either to make sure that it would produce a reliable results against load tests results or to calculate Nγ values using different methods than Terzaghi. Either way, Terzaghi equation is the most popular used equation by engineers in practice and by researchers. For example, a research has been done by Felipe Alberto in 2000 where he tested Terzaghi, Hansen, Meyerhof equations experimentally. He used circular plate loading testing method and compared the results of bearing capacity and bearing capacity factors. He found out that Terzaghi's equation produced very close values to the actual ones and therefore it is the most safe equation compared to the other ones.

Another research was done by D.Y. Zhu in 2003 to determine the bearing capacity of shallow foundations without using superposition approximation. In this paper the author has proven that the bearing can be estimated to an acceptable degree of accuracy without using the superposition assumptions. Terzaghi equation is used to express the bearing capacity but he used the critical slip field method to calculate Nγ which is dependent on the surcharge ratio and the internal angle of friction. One of the conclusions of this paper is that the values of the Nγ calculated using the superposition method is with 10% error on the safe side. By using the critical slip field method, the author was to reduce this percentage to 7% on the safe side.

Neural Artificial Network has been used in research as well to predict the bearing capacity of shallow foundations. Results from this ANN have been compared with theoretical values obtained from Terzaghi and it was found that Terzaghi's equation had a high correlation with values produced by the ANN.

## 3.1 EXPERIMENTAL PROCEDURES

The experimental procedure results used for this work were presented previously by Felipe Alberto, 2000 for his master thesis. In his experimental work, he tested cohesionless soil properties that included angle of friction, unit weight, relative density, and grain size distribution. His research focused on the bearing capacity of shallow foundation in sandy soil. For this purpose plate loading tests were perfumed in a lab and reported in his paper. Different sizes and shapes were used to test the current theories of bearing capacity and compared to experimental results to arrive at the most accurate theory. The tested theories included Terzaghi’s, Meyerhof’s, Hansen’s and Vesic’s. The results of this experimental tests will be reported to provide a better understanding of the aim of this paper.

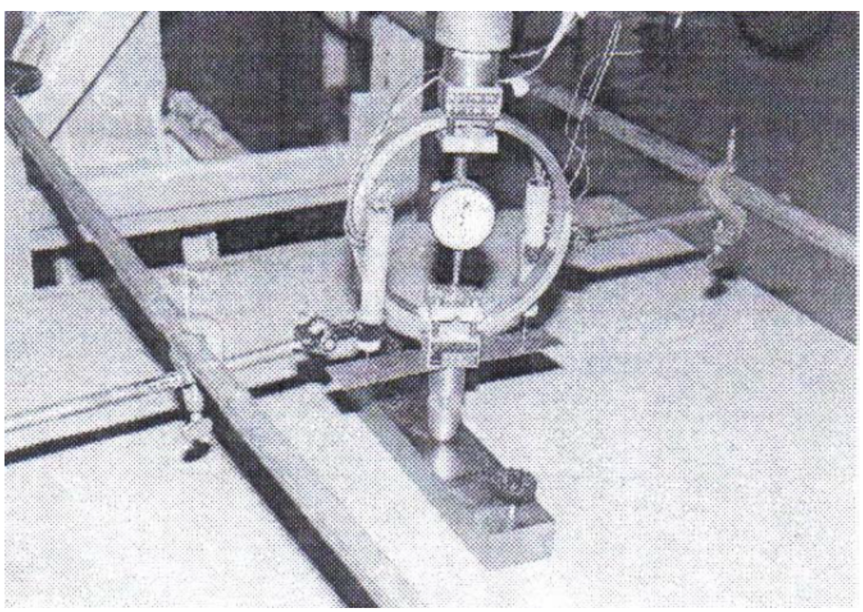


Figure 1: Apparatus set for plate loading test on sand (Felipe Alberto, 2000)

## 3.2 NUMBERICAL PROCEDURES

Based on the experimental procedure in the previous section, Terzaghi's equation was found to provide the most accurate estimation of the bearing capacity for shallow foundation. Also, this conclusion is supported by another research using ANN technique. So, the purpose of this section is to study the reliability of this theory based on statistical parameters that were obtained from previous section to carry out the reliability analysis process. These statistical parameters were the mean and the standard deviation of the random variables. Due to the lack of complete understanding of the soil behavior, the random variable were chosen to cover all sources of variability in the soil and in Terzaghi's equation. It is worthwhile to mention Terzaghi's equation for strip foundation in this section.

$$q_{ult} = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma$$

Based on this equation the random variables were the angle of friction, unit weight of soil, depth of the foundation, diameter of the foundation, and the bearing capacity factors. The bearing capacity factors were chosen as random variables because they are a function of the angle of friction which is a random variable. Therefore, it was deemed necessary to consider them as random variable. The cohesion is not included because the tests were carried out on sandy soil. Due to the small number of test data for unity weight and angle of friction, Monte Carlo simulation technique was used to generate 100000 iterations based on the statistical parameters for the angle of friction and unit weight. For each of the generated values of the angle of friction, the Terzaghi's bearing capacity factors were calculated. Then the mean and standard deviation for each bearing capacity factor were also calculated to generate a new values for the bearing capacity factors by applying Monte Carlo simulation. This was done by writing a Matlab code along with using Excel to speed up the process. After using Monte Carlo simulation, the distribution type of the random variables was found to be normal. The process of Monte Carlo Technique used is exactly as outline by Reliability of Structures (Collins &Nowak, 2000).

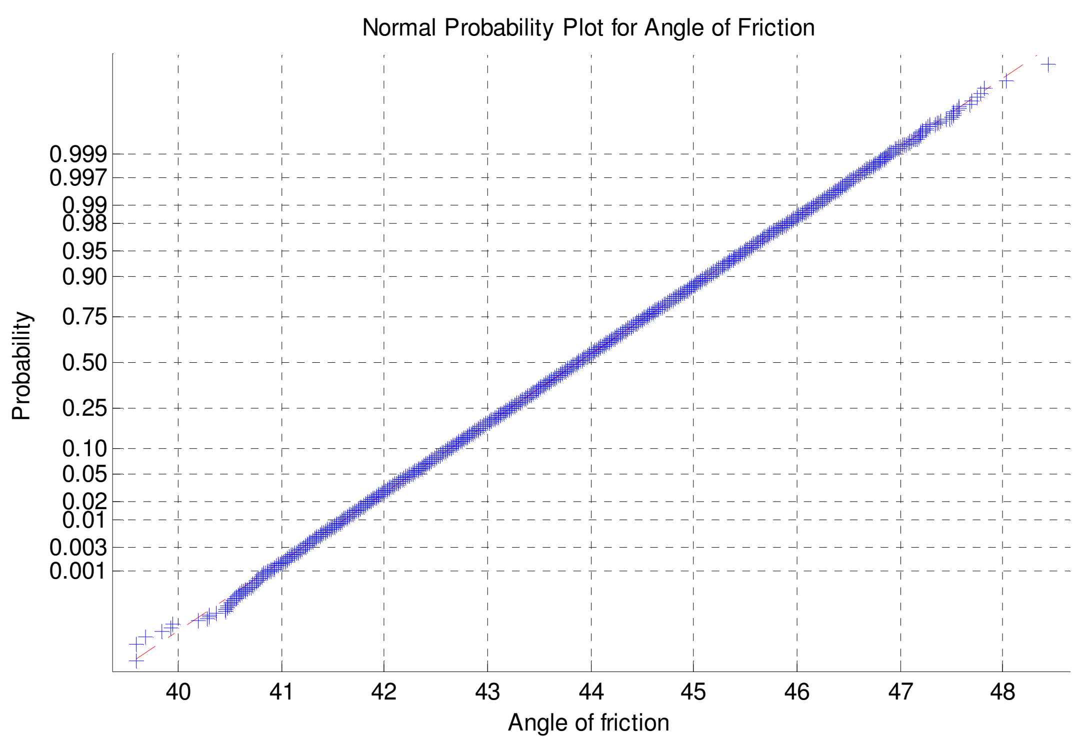



Figure 3.1 : Probability plot for Angle of Friction

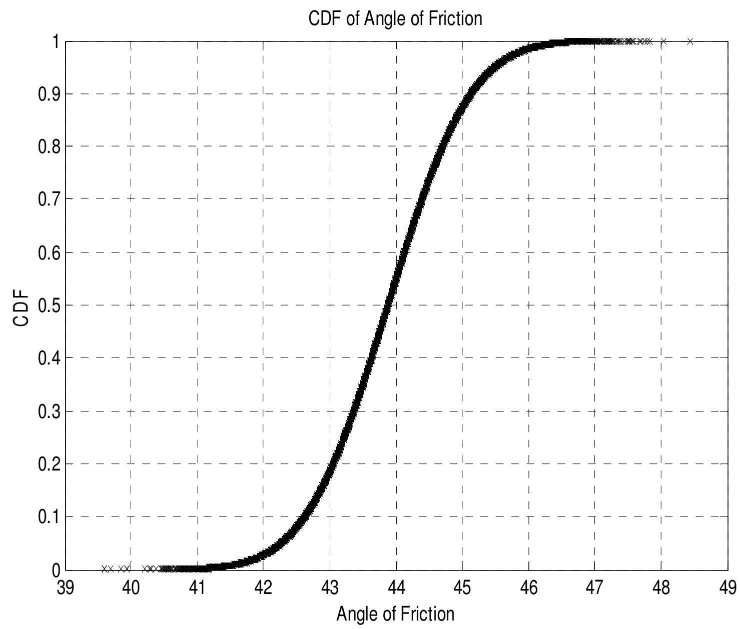



Figure 3.2: CDF of Angle of Friction

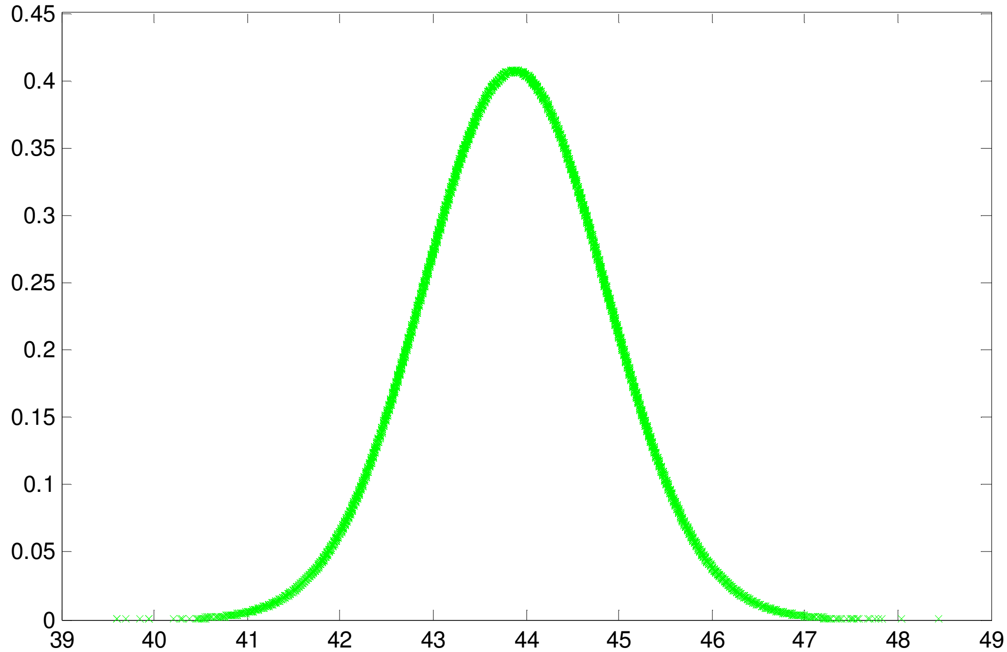



Figure 3.3: PDF of Angle of Friction

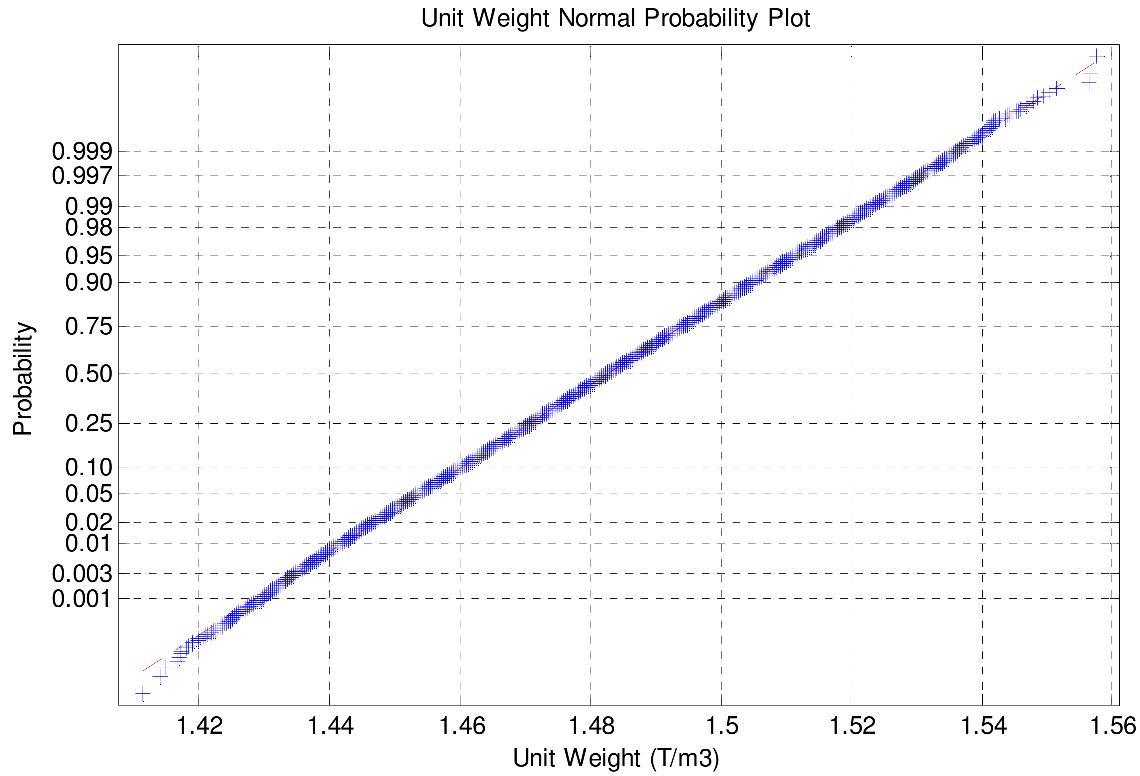



Figure 3.4: Unit weight probability plot

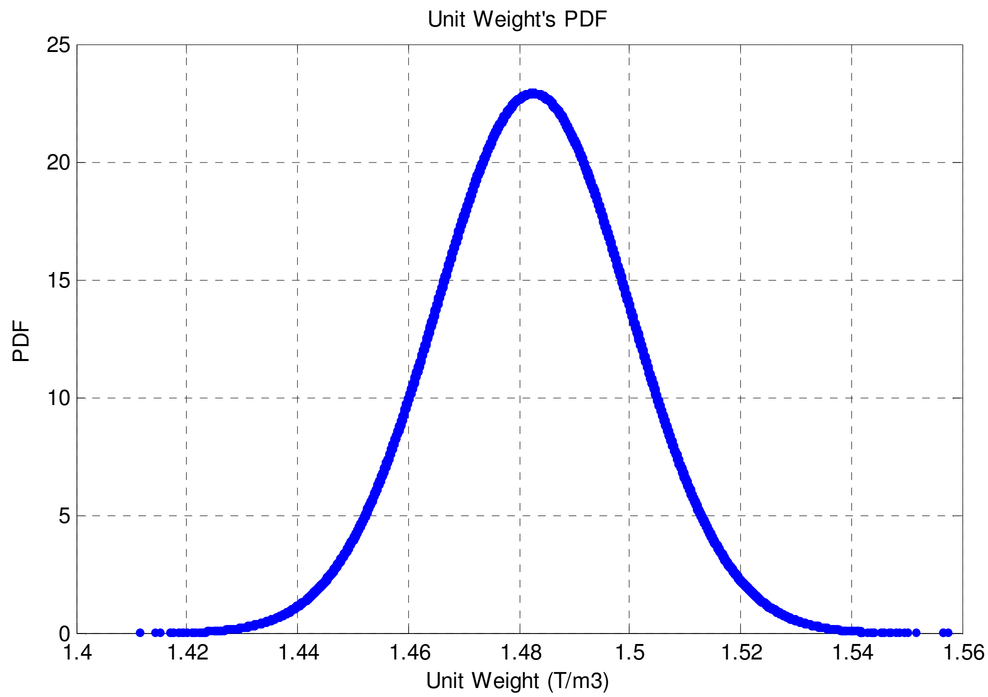



Figure 3.5: Unit Weight's PDF

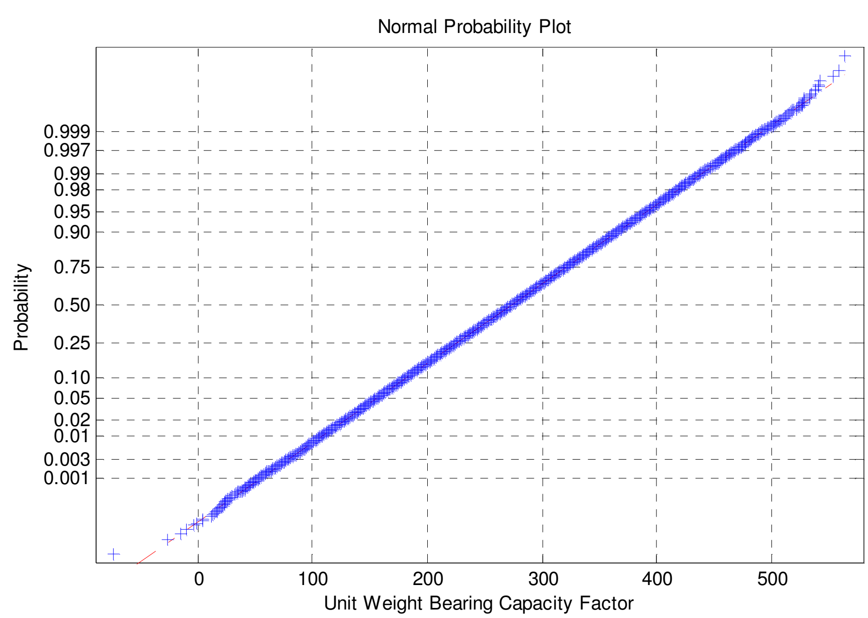



Figure 3.6: Ny Normal Probability Plot.

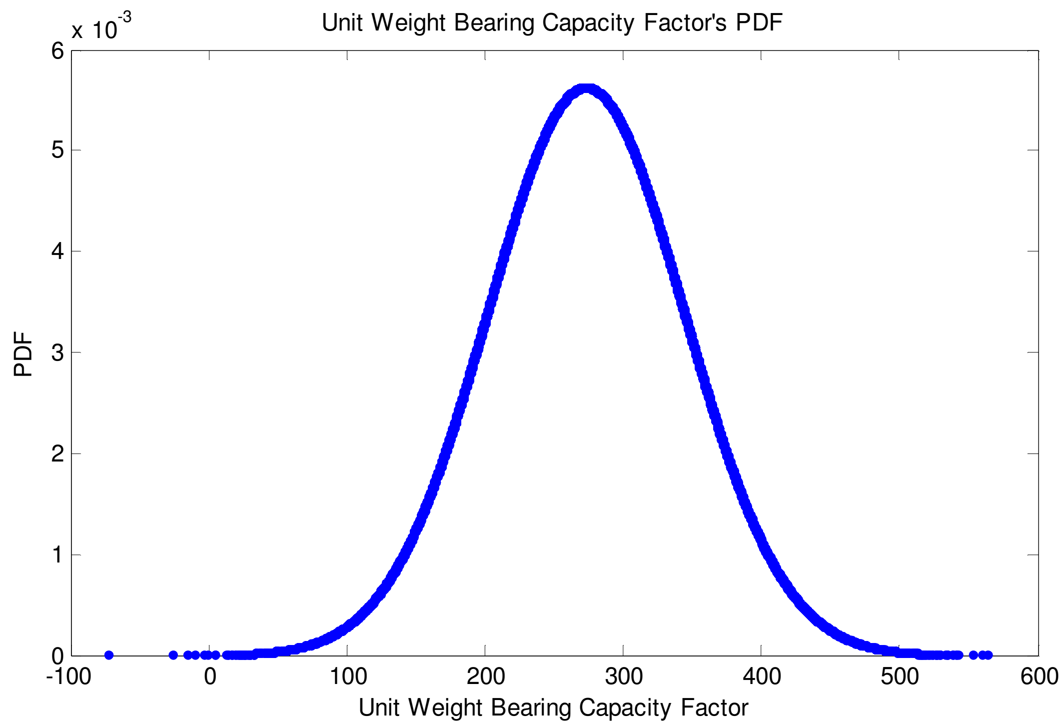



Figure 3.7: Ny's PDF

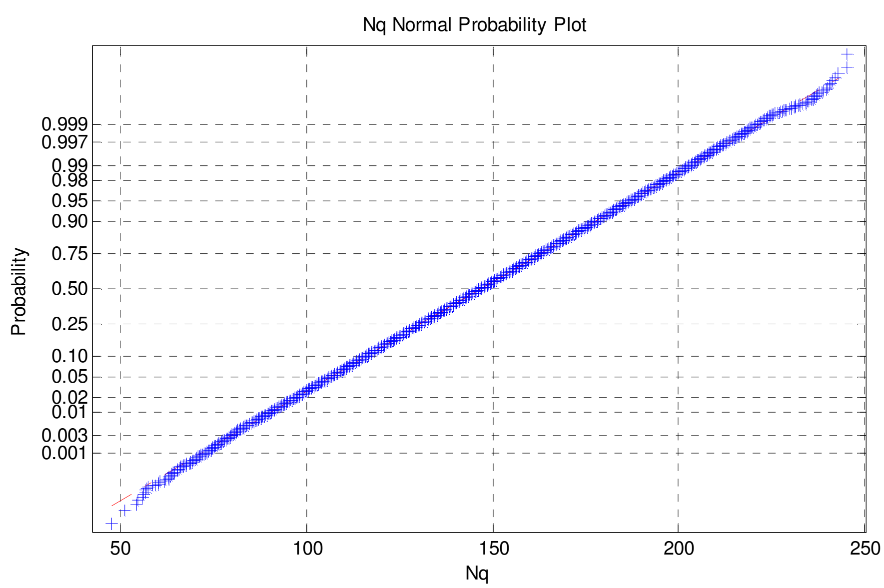



Figure 3.8: Nq Normal Probability Plot

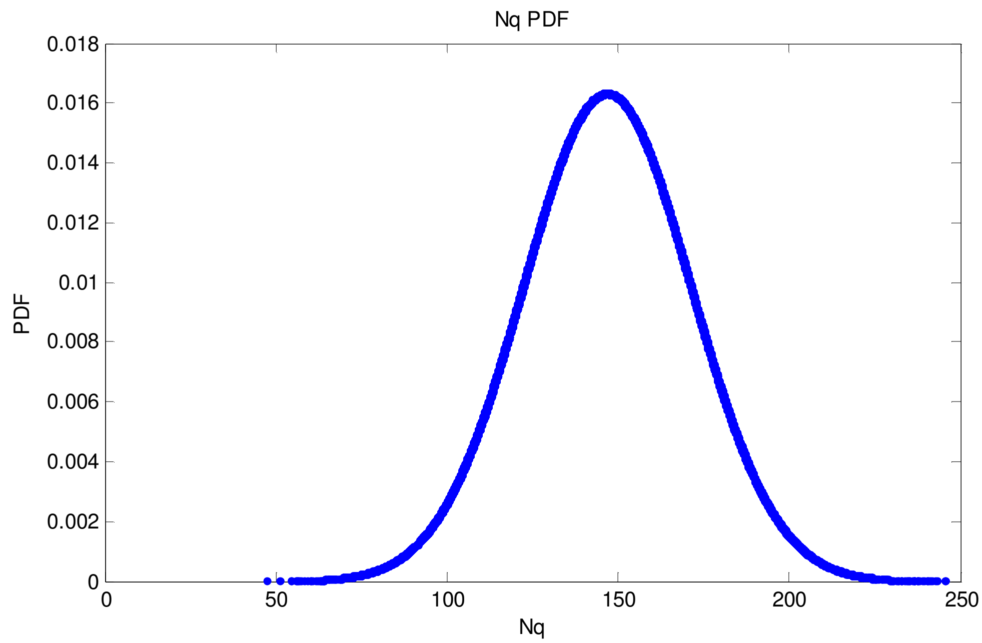



Figure 3.9: Nq PDF

As for the footing width and depth, these random variable were different for each design and were calculated based on the tolerable limits of practice. This limit is 15 cm or ½ foot. For each design case, a value for the width and depth was obtained in the predesign step. Then these values were used to generate random numbers +/- 15cm of these values to be used in the calculation of the bearing capacity.

# CHAPTER 4. CALCULATION OF RELIABILITY INDEX

The aim of this section is to calculate the reliability index of bearing capacity according to Terzahgi's theory under different loading for the given soil conditions, as well as to propose a resistance factor that would indicate a reliability index of 4 using the LRFD load factors of 1.2DL+1.6LL. For this purpose the shallow foundation shapes considered were circular, square and rectangular. The first order second moment method was used to calculate the reliability index for the different shapes of shallow foundation. According to this method the reliability index is:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

Where :

β = the reliability index.

$\mu_R$ $= the\ mean\ value\ of\ the\ resistance.$

$\mu_Q$ $= the\ mean\ value\ of\ the\ load\ effect.$

$\sigma_R^2, \sigma_Q^2 = the\ variance\ of\ the\ resistance\ and\ the\ load\ effect\ respectively.$

The resistance in this equation is represented by the bearing capacity of the soil which is composed of the angle of friction, unit weight, bearing capacity factors, depth, and width of the footing as random variables. The resistance for all the design cases followed a normal distribution as well as the loads. The loads were represented by the bias factors for dead load and live load as well as the coefficient of variation.

| Load Type | Dead Load | Live Load |
|---|---|---|
| Bias (λ) | 1.05 | 1 |
| COV | 0.1 | 0.18 |

Table 4.1: Live & Dead load data

The nominal value for the dead and live load was obtained from the design cases. Then the mean was calculated from the definition of bias as the nominal value over the mean. The standard deviation was calculated by multiplying the mean by the coefficient of variation. Then Monte Carlo simulation was used to generate random values for the dead and live load based on their mean and standard deviation. In the geotechnical design the width of the foundation is calculated from:

$$\frac{q_{ult}}{FS} = \frac{P+Wf}{area}$$

Where:

qult= the ultimate bearing capacity of the soil

P= dead load + live load

Wf= the weight of the foundation = depth*area*24KN/m3

FS= factor of safety

The ultimate bearing capacity is divided by the factor of safety which is chosen to be 3 and kept the same for all design cases. By setting these two parts of the equation equal, the width of the footing can be calculated. The depth of the foundation is assumed to be 25% of the width to keep the Df/B<1 as a condition for shallow foundation. After calculating the width, random numbers were generated in the range of plus or minus 15 cm of the width. Same was done for the depth of the footing. Finally, the ultimate bearing capacity was calculated for each value of the random variable as mentioned before. As a result the reliability of each case was obtain by applying the first order second moment method.

The previous procedures were done when the ASD design method was considered. However, when considering the LRFD design method along with the chosen factors of dead and live load, the process is different. In the LRFD case the load, factors are applied at first.

$$P = 1.2DL + 1.6\ LL$$

Then, the ultimate bearing capacity is multiplied by a resistance factor.

$$\emptyset * q_{ult} = \frac{P + Wf}{area}$$

Where:

□= resistance factor

qult= the ultimate bearing capacity of the soil

P= 1.2dead load + 1.6live load

Wf= the weight of the foundation = depth*area*24KN/m3

The resistance factor values varies between 0 and 1. In order to calculate the resistance factor that would result in a reliability index of 4, different iteration were done by increasing the resistance factor by step size of 0.1 using matlab code for this purpose. Then the proposed resistance factor that yielded a reliability index of 4 were averaged out and final resistance factor corresponding to the foundation shape is presented.

# CHAPTER 5. RESULTS AND DISCUSSION

## 5.1 RESULTS

### <u>Circular Footing</u>

**<u>Design Case I:</u>** **DL= 400 KN, LL=300 KN**

**<u>ASD:</u>**

Based on ASD design method, the reliability index for this design case that corresponds to a SF = 3 is $\beta = 2.844$.

The limit state function is: $g = R - Q$

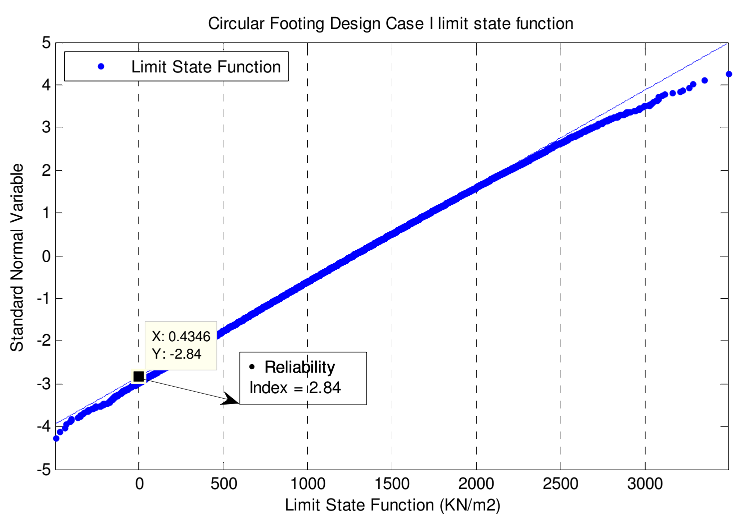



Figure 5.1: Limit State Function Design Case I ASD Method

However, when considering LRFD design method the reliability index is dependent on the resistance factor. The limit state function which is linear limit state and it's reliability index is a function of the resistance factor.

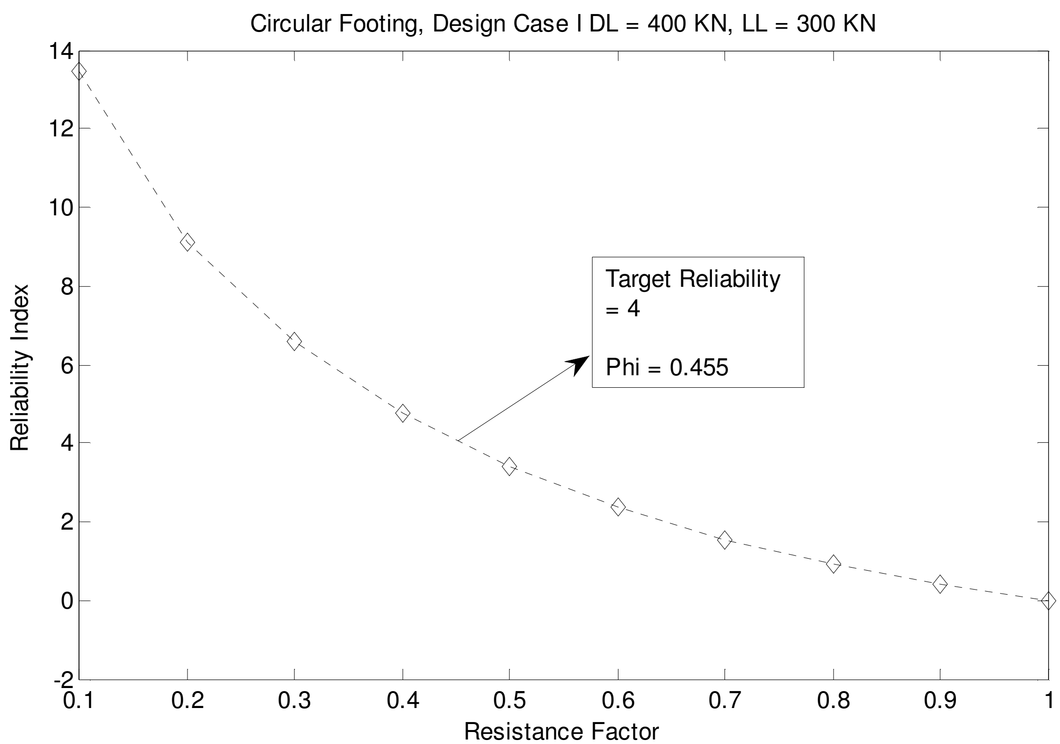



Figure 5.2 : Reliability Index Vs. Resistance Factor Design Case I LRFD Method.

The target reliability which is 4 chosen based on the target reliability of columns which is 4 as well. This target reliability index is obtained when the resistance factor  = 0.455.

**<u>Ciruclar Footing:</u>**

**<u>Design Case II:</u>** **DL= 500 KN, LL=280 KN**

**<u>ASD</u>**

FS = 3 ===➔ $\beta$ = 2.889

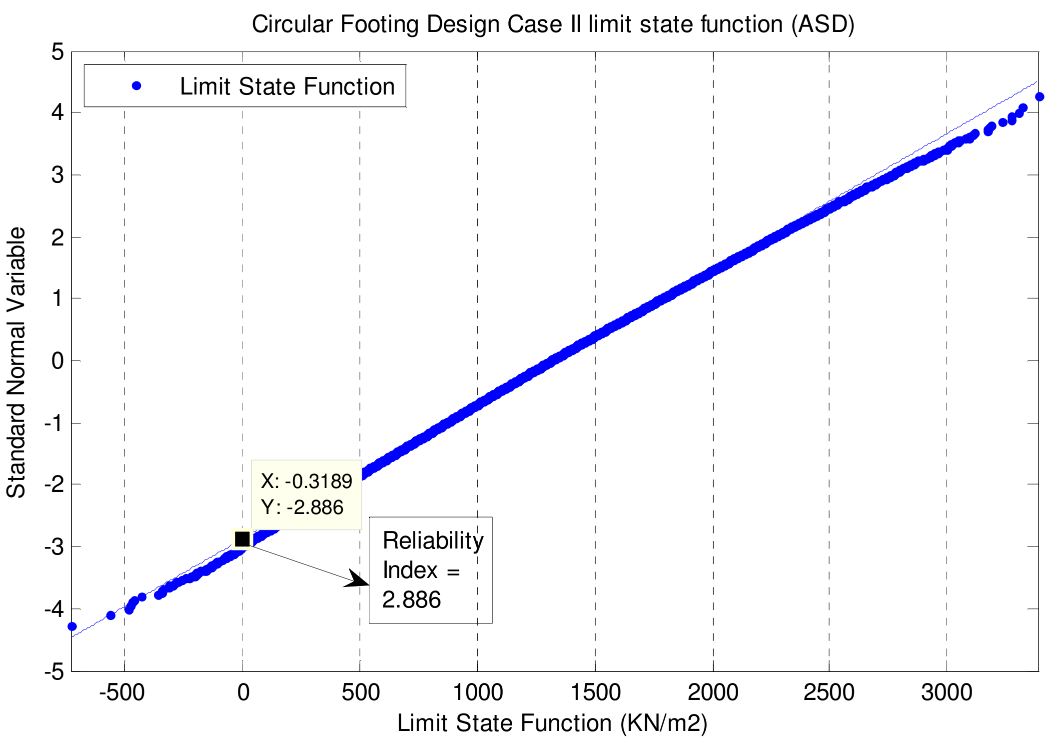



Figure 5.3 : Limit State Function Design Case II ASD Method

**LRFD:**

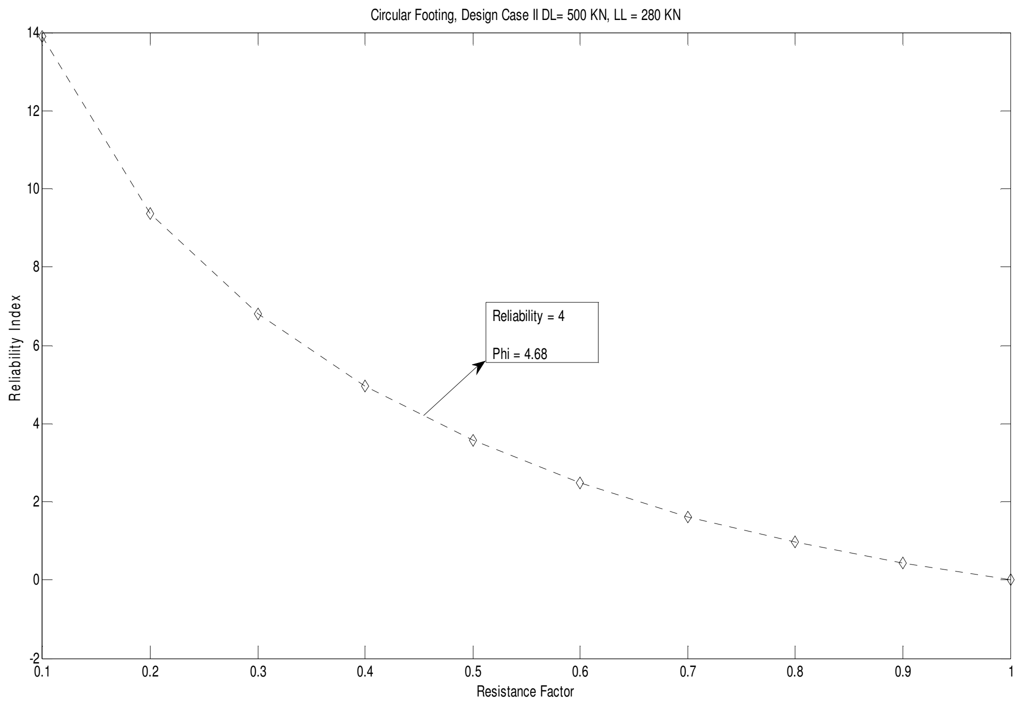



Figure 5.4 : Reliability Index Vs. Resistance Factor Design Case II LRFD Method

The resistance factor that corresponds to target reliability index of 4 in this case is □ = 0.468.

**<u>Design Case III:</u>** **DL= 300 KN, LL=150 KN**

**<u>ASD:</u>**

FS = 3 ===➔ $\beta = 2.696$

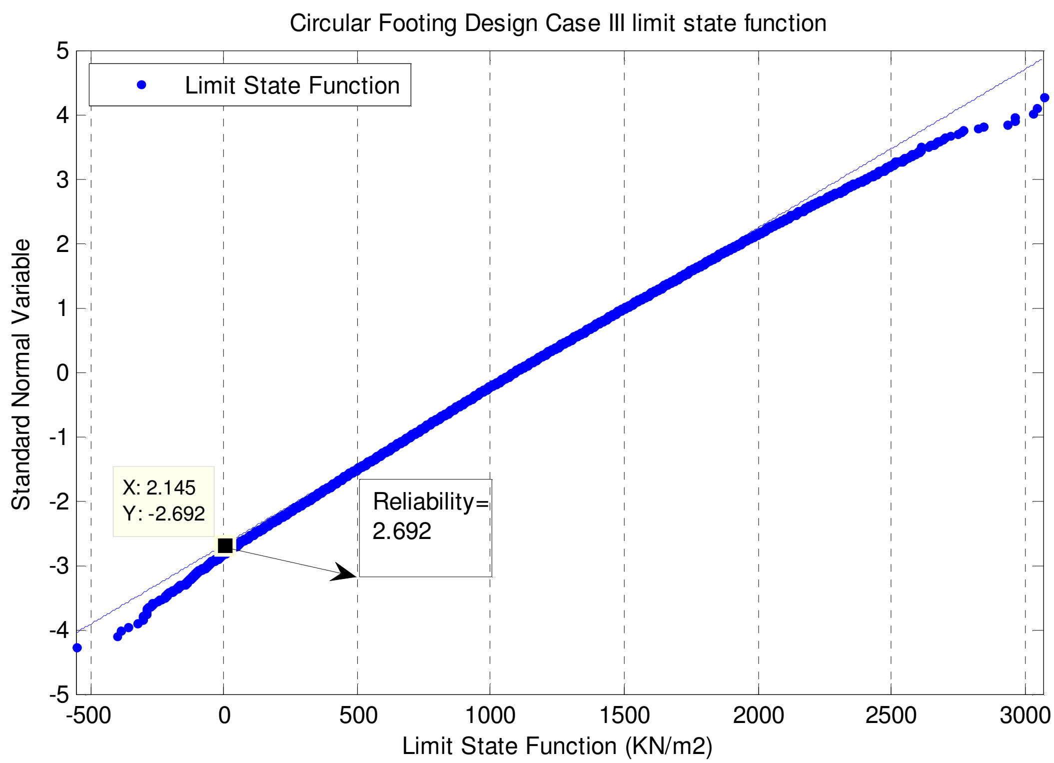



Figure 5.5: Limit State Function Design Case III ASD Method.

**LRFD:**

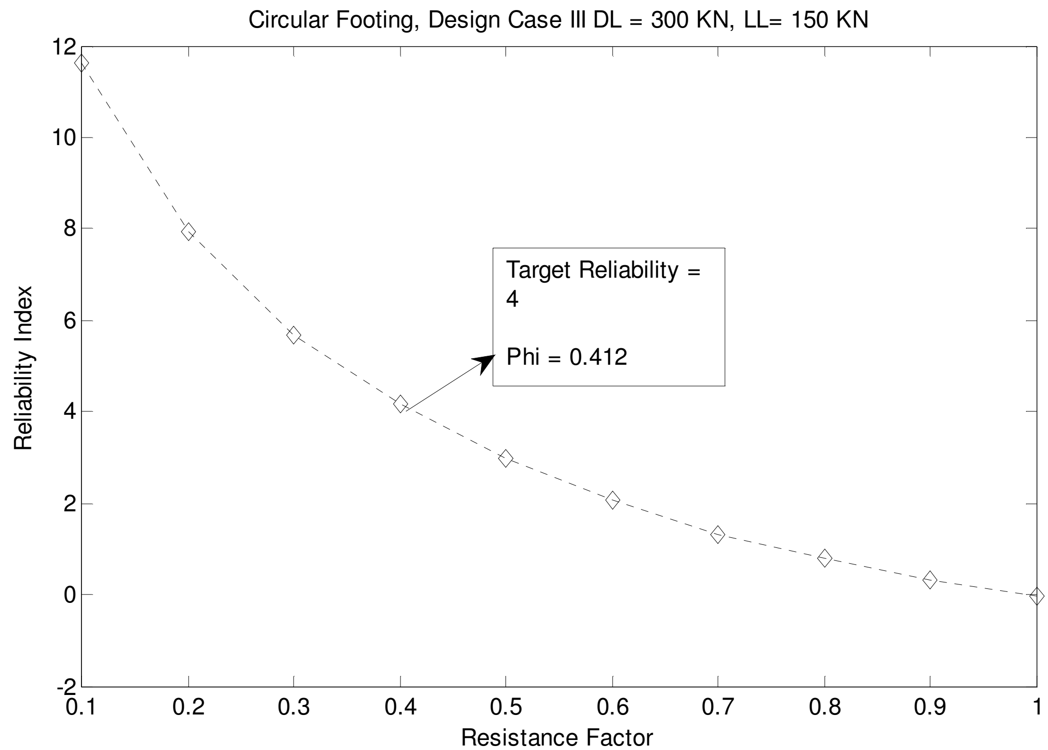



Figure 5.6: Reliability Index Vs. Resistance Factor Design Case III LRFD Method

**<u>Design Case IV:</u>** **DL= 400 KN, LL=100 KN**

**<u>ASD:</u>**

FS = 3 ===➔ $\beta = 2.76$

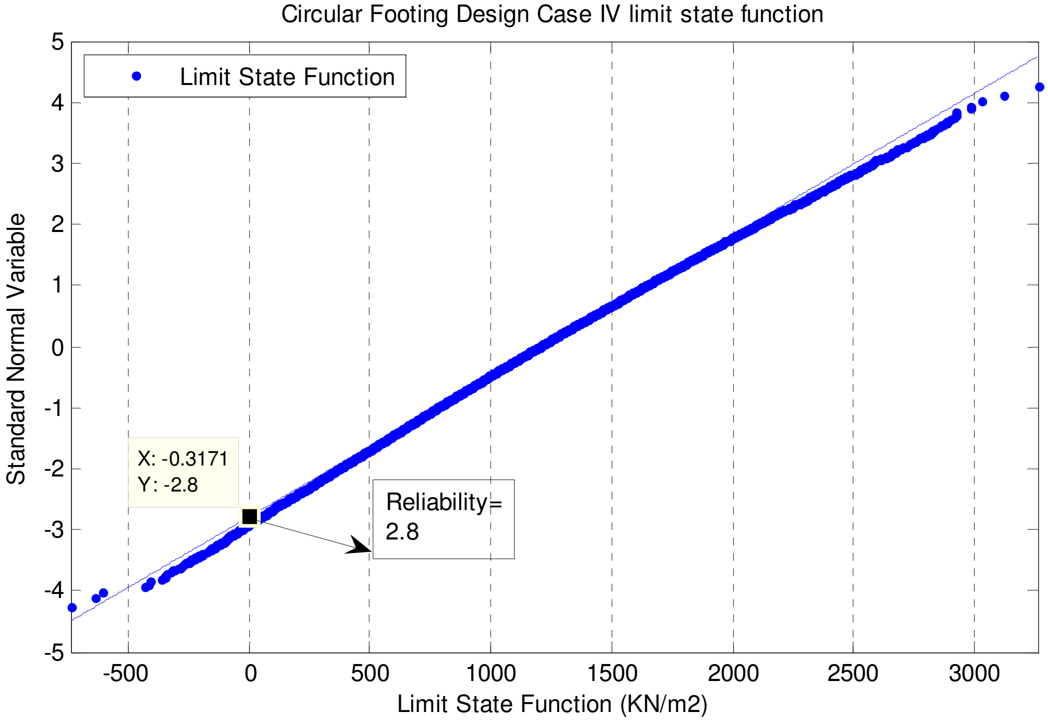



Figure 5.7: Limit State Function Design Case IV ASD Method.

**LRFD**:

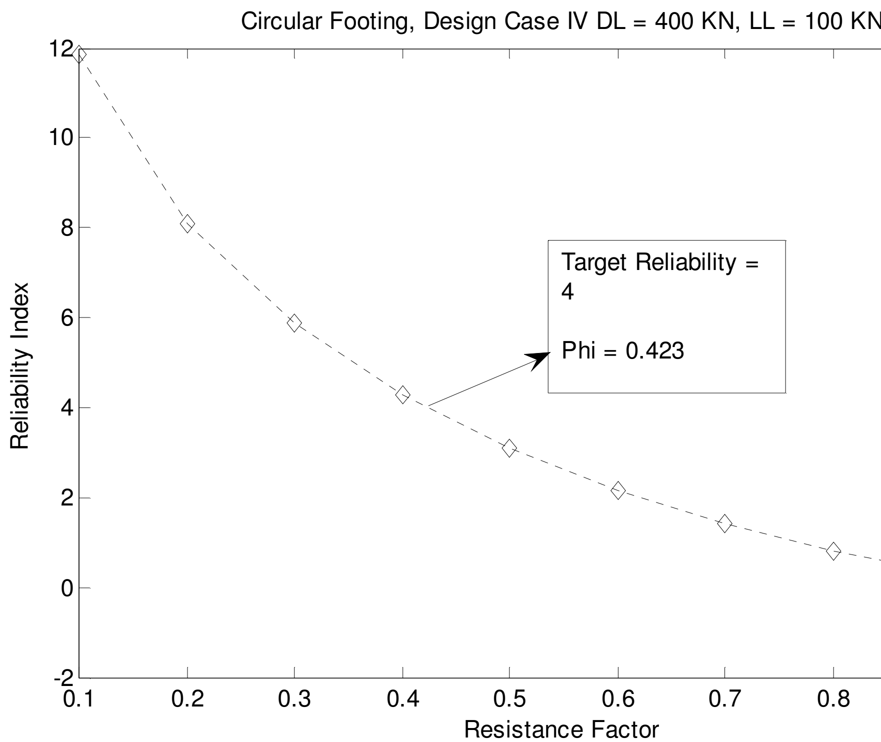



Figure 5.8: Reliability Index Vs. Resistance Factor Design Case IV LRFD Method

**<u>Design Case IV:</u>** **DL= 400 KN, LL=150 KN**

**<u>ASD:</u>**

FS = 3 ===➔ β = 2.77

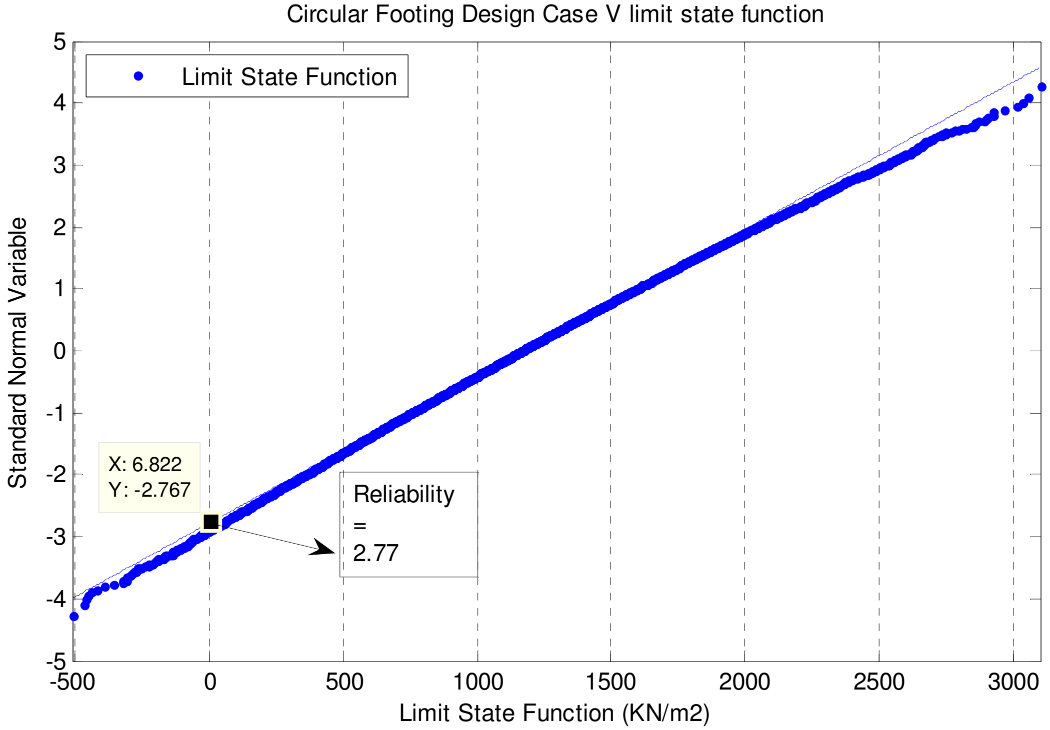



Figure 5.9: Limit State Function Design Case V ASD Method.

**LRFD**:

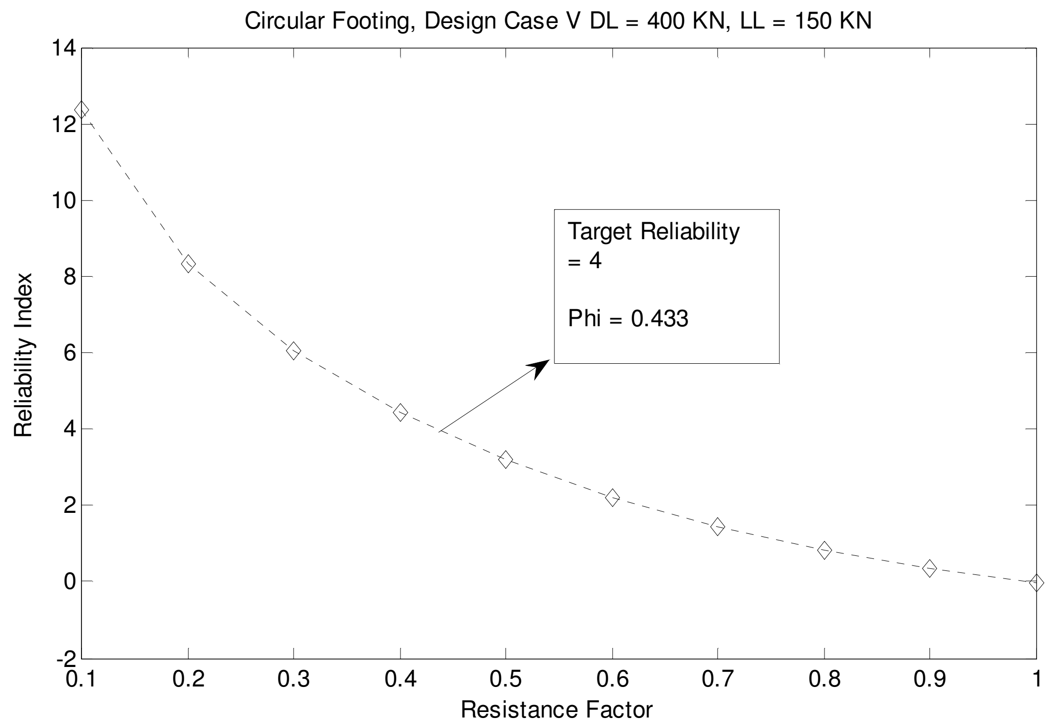



Figure 5.10: Reliability Index Vs. Resistance Factor Design Case V LRFD Method

## SQUARE FOOTING

### DESIGN CASE I: DL= 400 KN, LL=300 KN

### ASD:

FS= 3 ====➔ $\beta = 2.76$

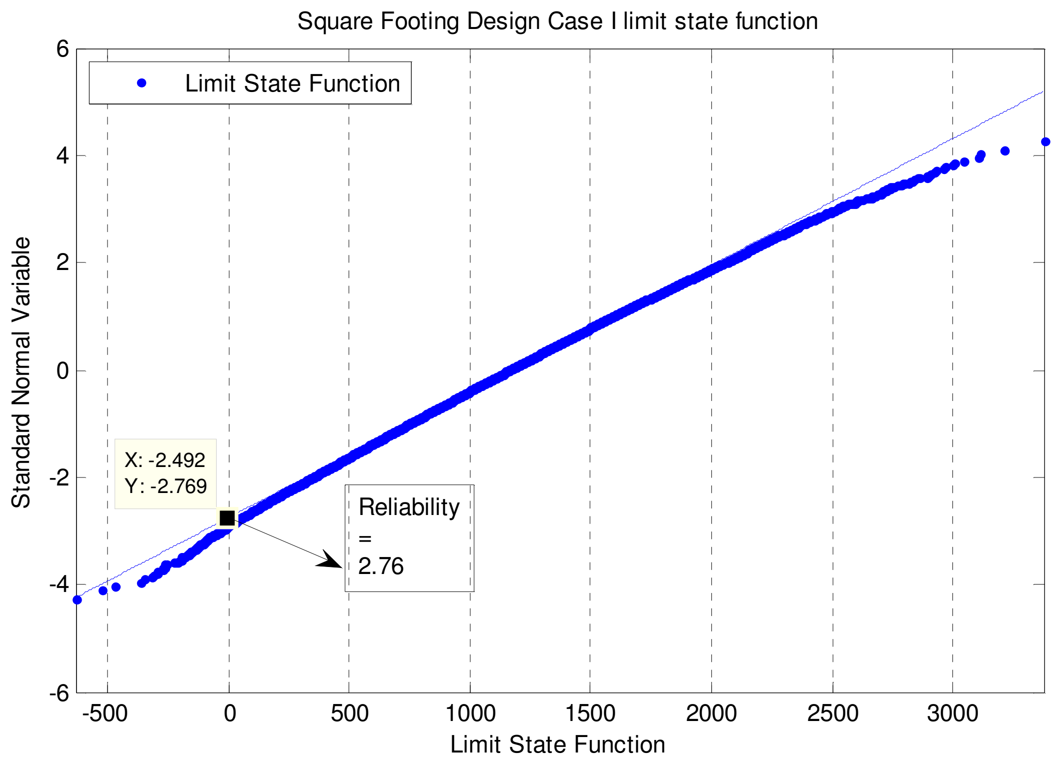



Figure 5.11: Square Footing Limit State Function Design Case I ASD Method.

**LRFD**:

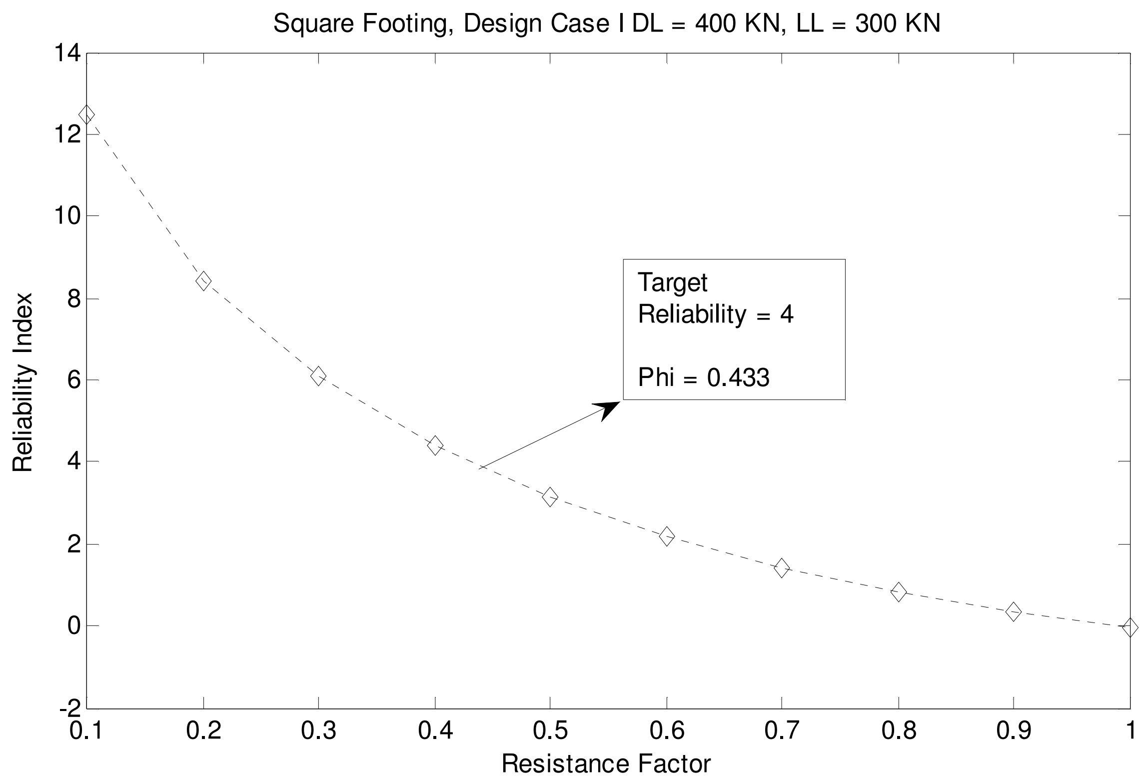



Figure 5.12: Reliability Index Vs. Resistance Factor Design Case I Square Footing LRFD Method.

**DESIGN CASE II:** **DL= 500 KN, LL=280 KN**

**ASD:**

FS= 3 ====➔ β = 2.809

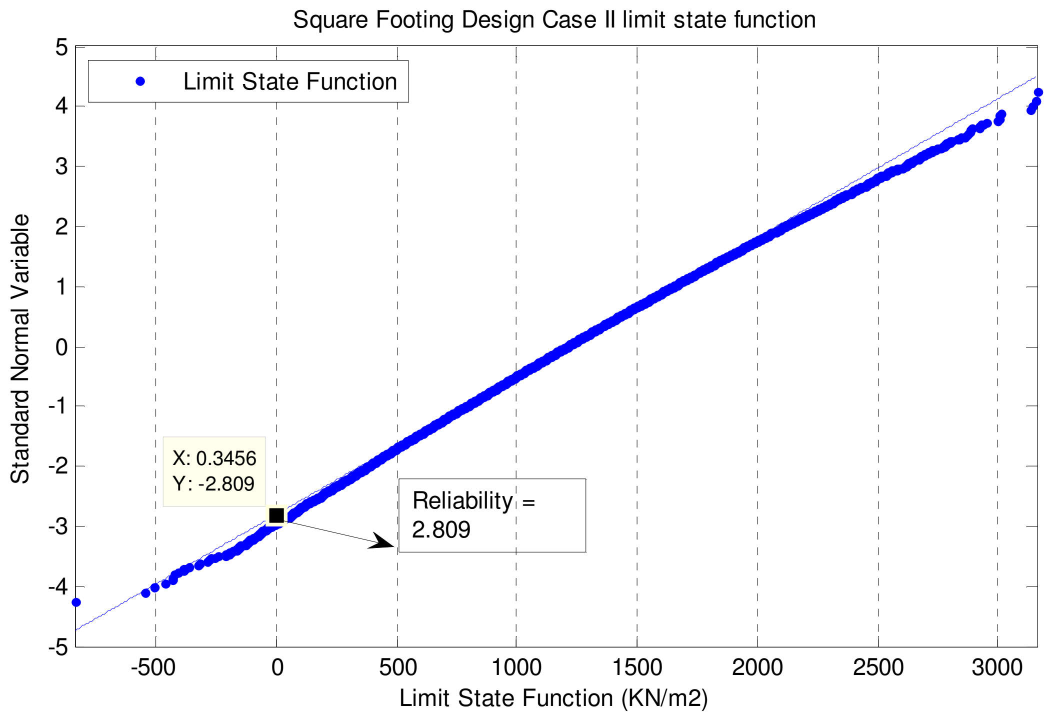



Figure 5.13: Square Footing Limit State Function Design Case II ASD Method.

**LRFD**:

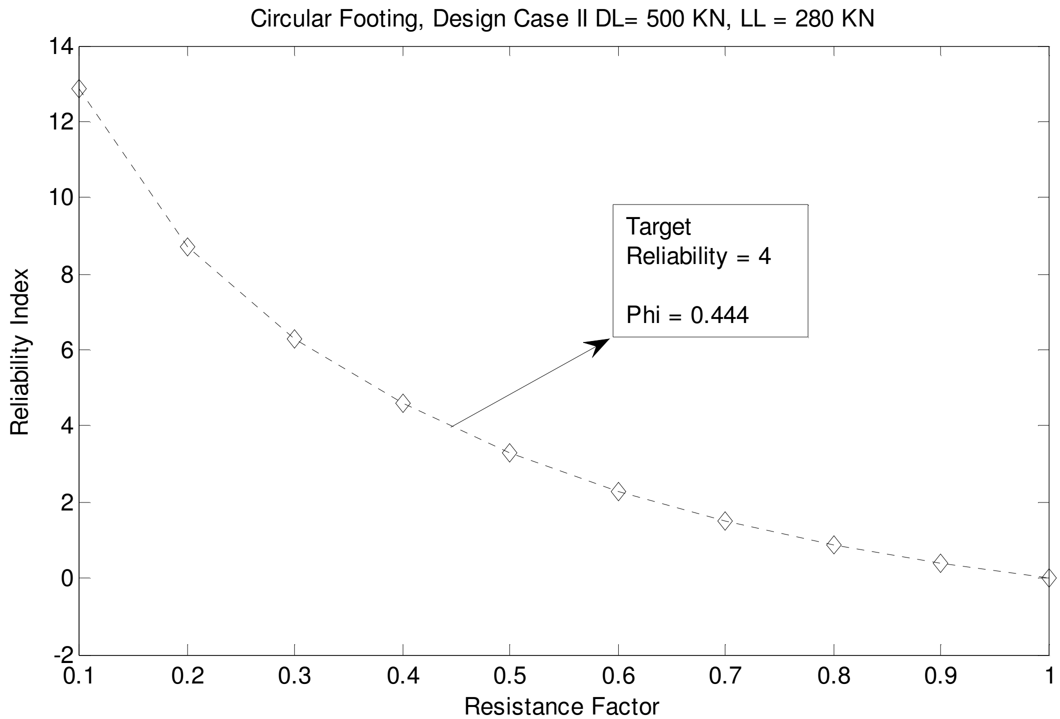



Figure 5.14: Reliability Index Vs. Resistance Factor Design Case II Square Footing LRFD Method.

**DESIGN CASE III:** **DL= 300 KN, LL=150 KN**

**ASD:**

FS= 3 ====➔ $\beta = 2.6$

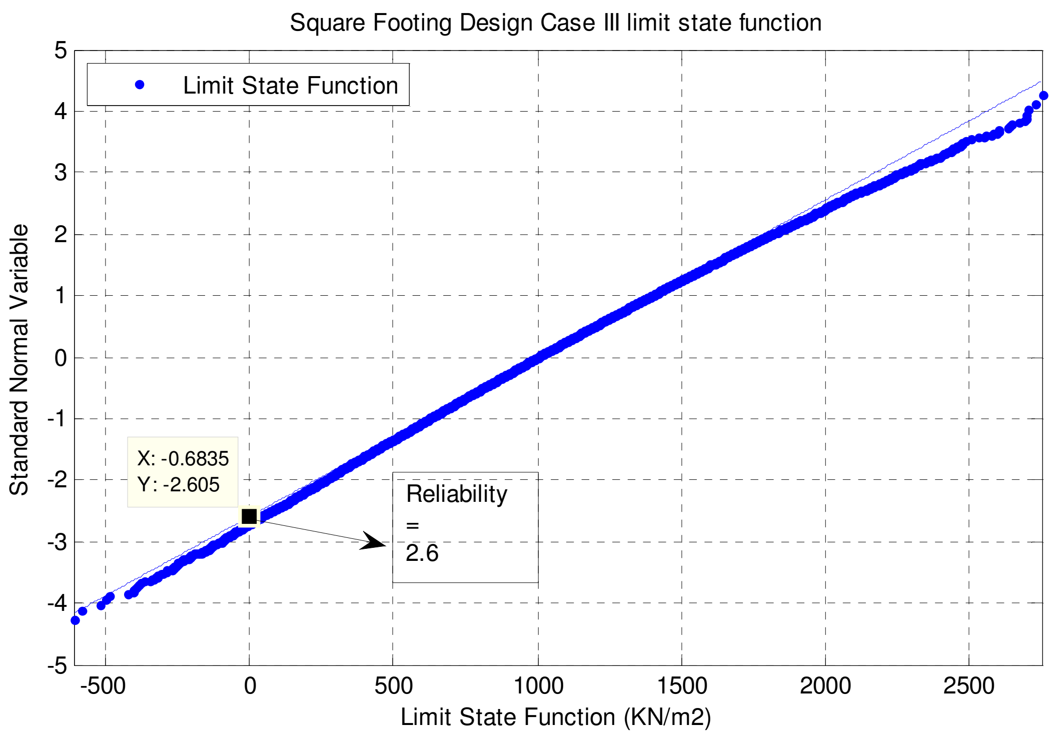



Figure 5.15: Square Footing Limit State Function Design Case III ASD Method.

**LRFD:**

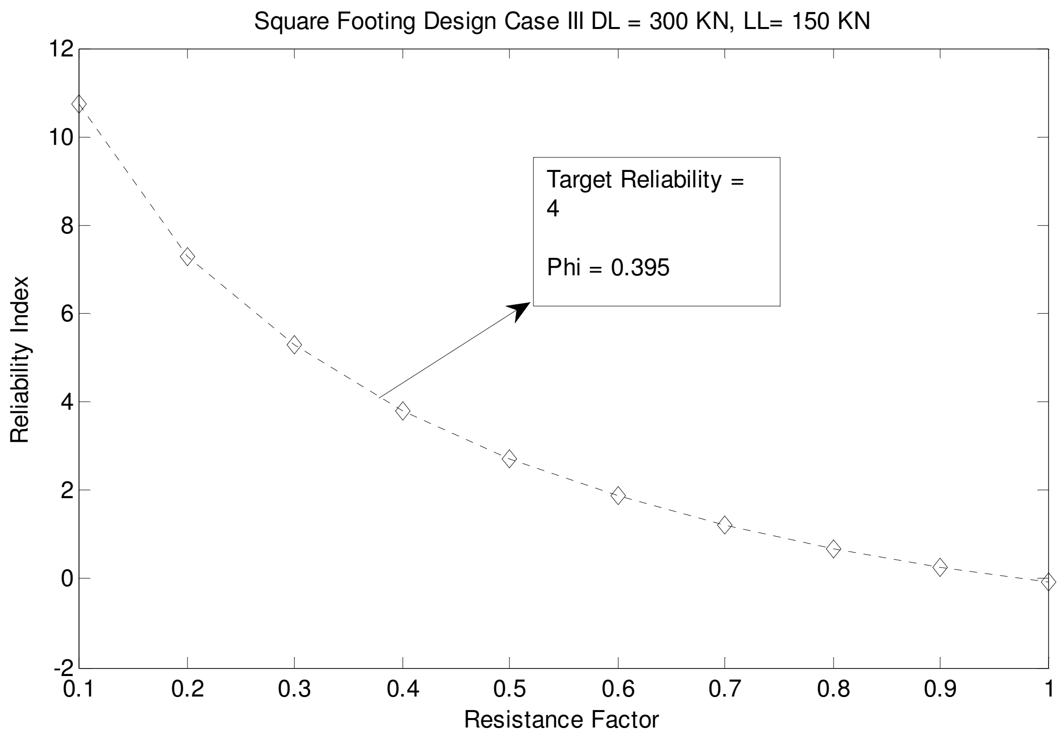



Figure 5.16: Reliability Index Vs. Resistance Factor Design Case III Square Footing LRFD Method.

**DESIGN CASE IV:** **DL= 400 KN, LL=100 KN**

**ASD:**

FS= 3 ====➔ $\beta = 2.65$

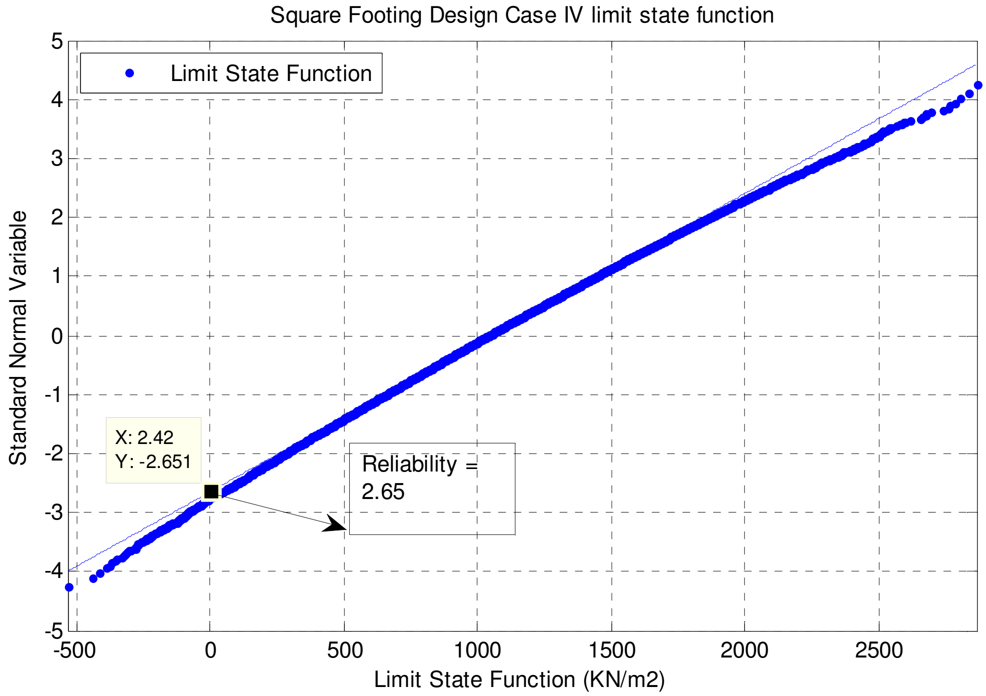



Figure 5.17: Square Footing Limit State Function Design Case IV ASD Method.

**LRFD:**

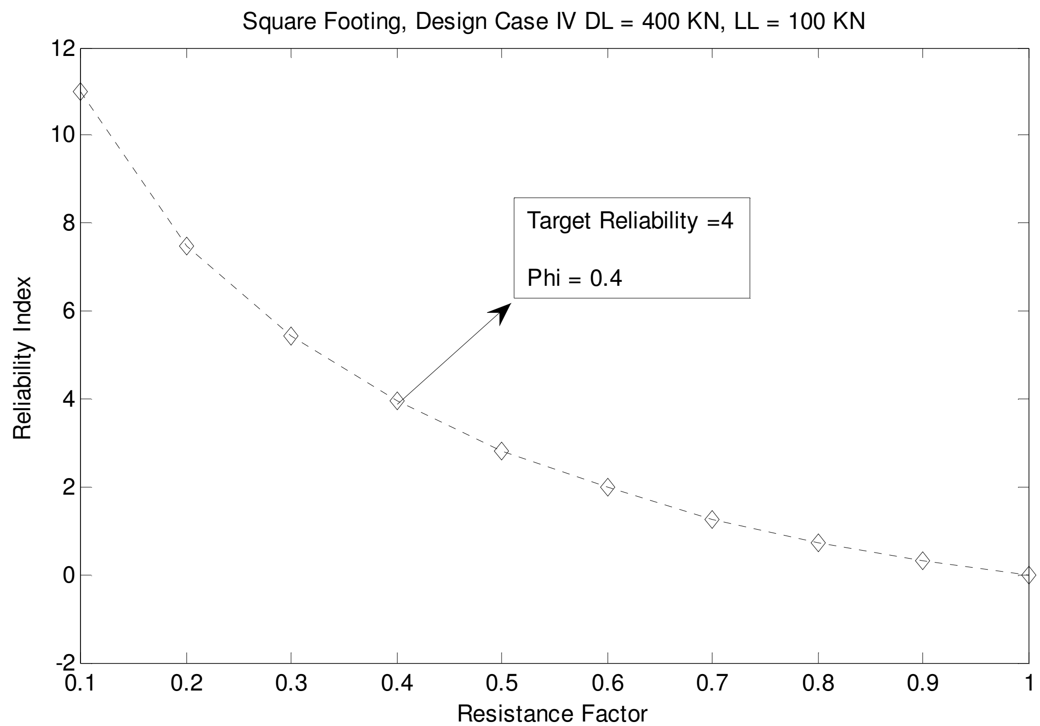



Figure 5.18: Reliability Index Vs. Resistance Factor Design Case IV Square Footing LRFD Method

**<u>DESIGN CASE V:</u>** **DL= 400 KN, LL=150 KN**

**<u>ASD:</u>**

FS= 3 ====➔ $\beta$ = 2.69

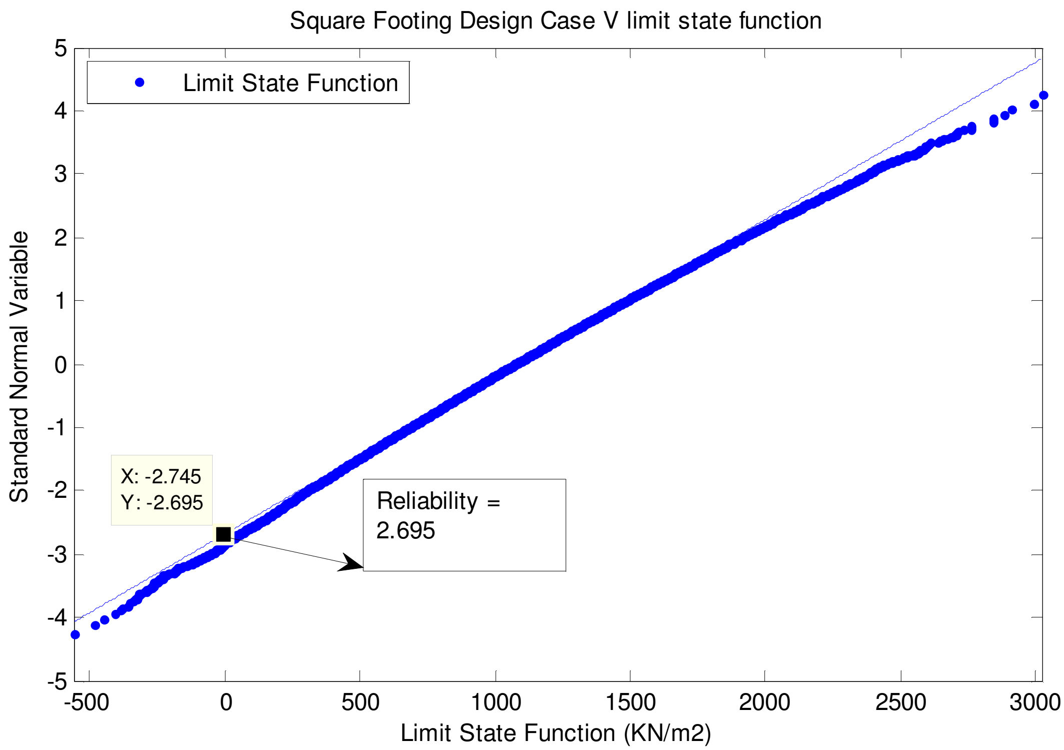



Figure 5.19: Square Footing Limit State Function Design Case V ASD Method.

**LRFD**

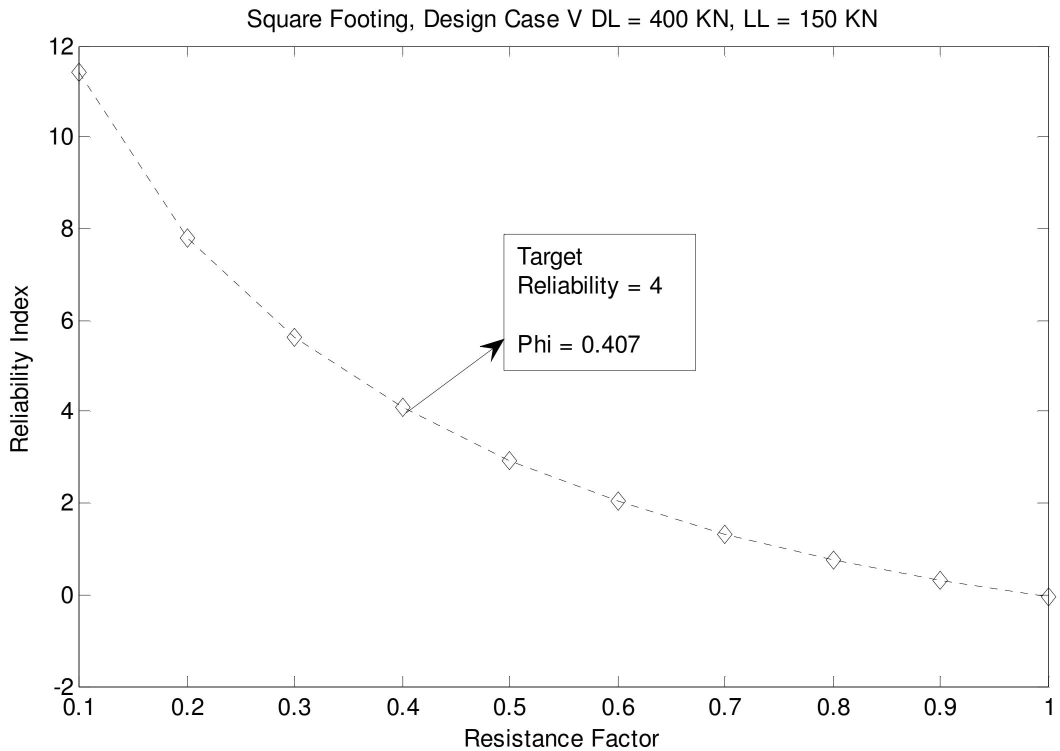



Figure 5.20: Reliability Index Vs. Resistance Factor Design Case V Square Footing LRFD Method.

## 5.2 DISCUSSION

Based on ASD method, which is currently adapted for geotechnical design, Terzaghi equation produce a probability of failure in the range of (0.0046-0.00248) for square footing. The probability of failure for circular footing was in the range of (0.0035- 0.0019).

| Design Case | Circular | Square |
|---|---|---|
| I | 2.84 | 2.76 |
| II | 2.89 | 2.8 |
| III | 2.69 | 2.6 |
| IV | 2.76 | 2.65 |
| V | 2.77 | 2.69 |
| Average | 2.79 | 2.7 |

Table 5.2: Reliability indices for shallow foundation

This is sufficient and produces a reliable results. However, because the foundation is the most important part of the structures, the probability of failure should be less or at least equal to the probability of failure for columns. Due to this reason, LRFD seems to be more appropriate for this purpose.

Five loading scenarios were chosen for circular and square footing. The idea was to study the resistance factor of each foundation shape that would result in a target reliability index of 4. The same loading conditions were applied to the two shapes to see if the shape of the footing will play a factor in determination of the resistance factor. Also, choosing different loading conditions would give an indication if the resistance factor is affected by the loading ratio between dead load and live. Hence, the different

| Design Case | Resistance Factor corresponding to $\beta = 4$ (Circular Footings) | Resistance Factor corresponding to $\beta = 4$ (Square Footings) | Loading Ratio LL/DL |
|---|---|---|---|
| Case I | 0.455 | 0.433 | 0.75 |
| Case II | 0.468 | 0.444 | 0.56 |
| Case III | 0.412 | 0.395 | 0.5 |
| Case IV | 0.423 | 0.4 | 0.25 |
| Case V | 0.433 | 0.407 | 0.37 |

loading ratios for different design cases. The results obtained were as follows :

Table 5.2: Resistance for different shapes of footings corresponding to Reliability Index of 4.

Starting with the Circular footing, the loading ratios were chosen arbitrarily. For each design case the resistance factors are different. However, the difference is in the range of +/- 5 % which is an accepted level of accuracy. This means that the shape of the footings did not play a role in the value of the resistance factor. The average of these resistance factors for circular footing is 0.44. When considering the square footings, the difference between the different resistance factors was also within the accepted level of accuracy which +/- 5% and the average was 0.41. The same thing could be said about the square footing, that the resistance factor in independent of the shape. The next step in the analysis is to change the loading ratio in certain range to study the range of the resistance factor. The uncertainty of the live load is greatest for smaller values of the LL/DL ratio and this influence of certainty is negligible for LL/DL > 4 ( Galambos, et al, 1982). For this purpose the starting ratio was chosen to be 0.5 and the upper limit for LL/DL was chosen to be 4. Beyond the ratio of 4 the loading ratio has negligible effect on the resistance factor (Ellingwood et al, 1982).

| LL/DL | Resistance factor that correspond to β = 4 (Square Footing) | Resistance factor that correspond to β = 4 (Circular Footing) |
|---|---|---|
| 0.5 | 0.3576 | 0.3775 |
| 1 | 0.3511 | 0.3677 |
| 1.5 | 0.3474 | 0.3629 |
| 2 | 0.3452 | 0.3594 |
| 2.5 | 0.3448 | 0.3574 |
| 3 | 0.3439 | 0.3549 |
| 3.5 | 0.3436 | 0.3548 |
| 4 | 0.3440 | 0.3544 |

Table 5.3 : Resistance factor Vs LL/DL ratio for Circular and Square Footings.

The purpose of changing the loading ratio for the same shape of footing is to study its effect on the resistance factor. Considering the circular footing, the values of the resistance factors are different within the acceptable level of accuracy. The average of this resistance factor for circular footing is 0.36. The values of the square footing are also within the accepted level of accuracy and the average is 0.34.

| Resistance Factor Average | Based on Shape | Based on LL/DL ratio |
|---|---|---|

| | | |
|---|---|---|
| Circular Footings | 0.44 | 0.36 |
| Square Footing | 0.41 | 0.34 |

Table 5.4: Average of the Resistance factor.

Based on the difference between resistance factors and the accepted level of accuracy of +/- 5 % for each shape and design case, it is safe to say that resistance factor is dependent on the loading ratio. It is worthwhile to mention once again that these tests were performed on sand. Therefore, these findings are related to sand only. In order to achieve a single resistance factor for shallow foundation on sandy soil, the average of the resistance factors based on LL/DL is the only one considered. The average based on the shape is disregarded because it is bigger than the average based on LL/DL, and because choosing a lower resistance will yield a reliability index higher than 4 and will satisfy all the loading ratios. Therefore, the proposed resistance factor is 0.35 for LRFD of shallow foundations bearing capacity on sand based on plate loading test.

# CHAPTER 6. SUMMARY AND CONCLUSION

Bearing capacity of shallow foundation is one of the most challenging problems for engineers. The difficulty comes from multiple sources of variability and uncertainty. There is an uncertainty in the live load and some in the dead load. Soil properties including: unit weight, cohesion, and angle of friction represent sources of variability in the determining bearing capacity. The current theories used in practice only estimate bearing capacity and does not give an exact value for it because of these sources of variability. Currently, there are Terzaghi, Meyerhof, Vesic, and Hansen theories for dealing with this problem. Based on previous research done by Felipe Aberto in 2000 on bearing capacity of shallow foundation on sand using plate loading tests, Terzaghi equation is the most close estimation tool to the real value of bearing capacity.

In this work, using the lab tests results published by Felipe Alberto, the reliability analysis done was performed to evaluate the Terzaghi's theory. The aim was to evaluate the reliability index of the ASD method that is currently adapted in geotechnical design using Terzaghi's theory for bearing capacity on sand. As well as proposing a resistance factor for LRFD method that would yield a reliability index of 4. The reliability index is chosen to be 4 to match the reliability index of columns and because foundation is the most important part of any structure. Therefore, this reliability index mean a lower probability of failure and a safer overall structure.

The reliability analysis was done by considering the unit weight, angle of friction, loads, width, and depth of the foundation as random variables. The bearing capacity factors were considered random variables as well because they are functions of the angle of friction. In order to simulate enough data points, Monte Carlo simulation was used to generate these values. The dead and live loads were presented in terms of the coefficient of variation and bias factor. The depth of foundation was assumed to be 25% of the width and kept the same for all simulations. As far as the width, the current tolerance limit is +/- 15 cm. Based on the initial step of geotechnical design a nominal value for the width was obtained and random numbers were generated in the range of +/- 15 cm of the nominal value. Same procedure used for the depth of the foundation.

After completing the simulation steps, a reliability index for the ASD method was calculated for each design case. Five design cases were considered for square and circular footings. In order to find a resistance factor that corresponded to reliability index of 4, the resistance factor was allowed to vary between 0.1 and 1 in step of 0.1. Graphs were obtained as result of this variation in the resistance factor and optimal value was obtain by interpolating between the resistance factor and the corresponding reliability indexes.

Five loading cases were chosen for each of the considered shapes to see if the shape of the foundation would play a role or not. It was found that shapes had no effect on the resistance factor and the obtained values were 0.44 for circular and 0.41 for square footings. In order to have a complete analysis, the loading ratio LL/DL was allowed to vary between 0.5 and 4. The average values obtained were 0.36 for circular and 0.34 for square footings. Since these values are lower than the values obtained when varying the shapes by more than 5%, it is recommended to consider the lower values. Considering these lower values would produce a safer design with lower probability of failure because the analysis covers practically all possible ranges of the LL/DL ratio. Therefore, the recommend value for resistance factor is the average of 0.34 and 0.36, which is 0.35.

As a recommendation for future research, clay should be considered in the analysis to arrive at a more representable value for the resistance factor. Also, other shapes of shallow foundations should be considered as well.

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